Estimating a Dynamic Game of Political Advertising

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Abstract

This paper studies the effect of political advertising on the outcomes of elections to the U.S. Senate between 2000 and 2018. I provide descriptive empirical evidence that candidates' advertising choices are driven by their own ideologies and political experience, as well as by their opponents' advertising and their standings in the polls. To gain further insights, I develop and estimate a dynamic game of political advertising with endogenous ad content, namely whether an ad is positive or negative. Parameter estimates show that incumbents have a substantial fundraising advantage over challengers. Positive advertising that similarly benefits challengers and incumbents reinforces this incumbency advantage, while negative advertising that disproportionately benefits challengers works against it. A counterfactual public campaign financing policy that eliminates incumbents' fundraising advantages decreases the incumbent reelection rate by 3.6 percentage points. Hypothetical temporary bans on negative advertising increase the incumbent reelection rate, but decrease ideological extremism among the challengers who replace them.

KEYWORDS: Dynamic Advertising, Negative Advertising, Endogenous Ad Content, Dynamic Models of Electoral Competition, Polarization, Incumbency Advantage, Competitive Elections.

1 Introduction

American political campaigns are some of the most expensive marketing campaigns in the world. Candidates for federal office spent over 8.5 billion dollars (Passwaiter, 2020) and aired over 6 million ads on broadcast television (Ridout et al., 2021) in the 2020 election cycle alone.

To what extent does all of this advertising influence election outcomes? Many advanced democracies regulate when and how much candidates can spend on advertising out of fear that wealthy megadonors and deep-pocketed special interest groups can buy elections with enough advertising (Karanicolas, 2012). Campaigns, too, believe their advertising can influence elections, with the average campaign spending over half its war chest on advertising and media consulting (Ridout et al., 2012; Sheingate, 2016).

But even the wealthiest and most well-connected candidates have limited time and money to win over voters. They must decide when and how much to advertise and which message to air to voters. In particular, one of the most important—and most controversial—decisions candidates make is running positive ads based only on their own merits versus negative ads that highlight their opponent's weaknesses. How effective a candidate's positive and negative ads are may depend on both their own background and policy platform, as well as their opponent's. These decisions must anticipate and account for their opponent's actions, and the importance and complexity of these decisions have given rise to a multi-billion dollar political consulting industry.

This paper studies the effect of political advertising on election outcomes in the United States. I focus my attention on general elections to the U.S. Senate between 2000 and 2018, which are in many ways an ideal setting for studying advertising. Unlike in commercial settings, there is no product price competition, changes in product characteristics, or shocks to consumers' incomes to potentially confound the effect of advertising on demand. Voters, unlike consumers, cannot easily monitor their representatives and their challengers in the way they can purchase a new brand or click a website link and experience the advertised product for themselves. Virtually all markets are duopolies between one Republican and one Democrat, and there is no entry or exit of candidates during the general election.

Using time-stamped data on candidates' television ad spending and negativity,[1] I provide descriptive evidence that candidates' spending on positive and negative ads are driven by their ideological platforms and incumbency. I also show that there are clear dynamic patterns to candidates' spending and negativity decisions over the course of the election, and that candidates strategically respond to their opponents' advertising and how they are doing in the polls.

To gain further insights into how candidates' strategic advertising decisions affect election outcomes, I develop and estimate a non-stationary dynamic discrete game-theoretic model of elections between two major-party candidates. In the model, forward-looking candidates continually observe their standings in the polls and decide how much to spend on positive and negative ads. The returns to positive and negative ads can vary with candidates' political experience and ideological platform, as well as with whether or not the opponent has begun airing negative ads. When making their advertising decisions, candidates account for their opponent's optimal dynamic behavior and their uncertainty over future polls. Advertising is costly, but has the chance to improve a candidate's standings in the polls, thereby increasing the probability she eventually wins the election and its officeholder benefits.

I build on Igami (2017) and rely on the model's finite time horizon and a sequential moves assumption to guarantee the model has a unique Markov Perfect Equilibrium. These assumptions also reduce computing the equilibrium to solving a sequence of effectively single-agent problems, keeping a full-solution estimator computationally tractable. I discretize candidates' spending and negativity choices, as well as the polling margins, and estimate the model by full-information

[1] Throughout this paper, I use the term negativity to refer to the fraction of a candidate's ad spending that is devoted to negative ads.

maximum likelihood using Rust's (1987) nested fixed point algorithm.

Parameter estimates show that the marginal returns to both positive and negative advertising are positive and economically meaningful. I also find that incumbents face a significantly lower fundraising cost than challengers. Whether or not advertising reinforces or works against this incumbent fundraising advantage depends heavily on the advertising content. I find that the average return to positive advertising is similar for both incumbents and challengers. If candidates could only run positive ads, advertising would reinforce the incumbent's fundraising and initial polling advantages. Negative advertising, however, is substantially more effective for challengers than for incumbents, and thus works against incumbents' electoral advantages.

My estimates also suggest other key differences in the nature of positive and negative advertising. The marginal returns to positive advertising decay more quickly than for negative advertising, and the effectiveness of a candidate's negative advertising increases once their opponent goes negative. More specific to political advertising, I find that candidates' ideological platforms also influence the effectiveness of their ads. Negative ads targeting more ideologically extreme candidates are more effective than those targeting moderates. Ideologically extreme candidates also run less effective positive ads compared to moderates and therefore tend to run more negative campaigns overall.

In light of these findings, I use the estimated model to conduct two counterfactual policy experiments to highlight how differences in candidates' opportunity costs of advertising and the effectiveness of their negative ads influence election results. In the first counterfactual, I simulate how eliminating incumbents' fundraising advantage affects election outcomes, as would be the case if campaigns were financed with public tax dollars. I find that eliminating incumbents' fundraising advantage drives down the incumbent reelection rate by 3.6 percentage points. However, ignoring incumbents' endogenous responses to their better funded challengers and the narrowing of the polls would have overstated the impact of the policy by at least 130 percent.

In the second counterfactual, I study how hypothetical restrictions on negative advertising influence election outcomes. Specifically, I simulate elections when both candidates are banned from airing negative ads in the final weeks of the election. I find that limiting negativity trades off less competitive elections and higher incumbent reelection rates with less ideological extremism among candidates elected to Congress. Negative advertising plays a key role in keeping elections between incumbents and challengers competitive and helps challengers make up for their fundraising disadvantage. However, negative advertising also facilitates the election of more ideologically extreme candidates who rely most heavily on negative advertising to get elected.

This trade off has direct policy implications. On one hand, existing work on incumbent accountability has found that incumbents respond more to their constituents when their reelection prospects are in danger (Ansolabehere et al., 2001; Griffen, 2006; Mian et al., 2010; Iaryczower et al., 2022). On the other hand, Congressional polarization has been linked to legislative gridlock (Binder, 2003), income inequality (McCarty et al., 2016), and worse policy responses to financial crises (Mian et al., 2014).

Taken together, my results show that political advertising *can* influence the outcomes of close elections–a finding which is underscored by the frequent narrow margins of Senate control over the past two decades. My findings also speak to the debate surrounding money in politics in the wake of *Citizens United* and *SpeechNow v. FEC*. By overturning longstanding campaign finance laws, these landmark judicial rulings paved the way for individuals, corporations, and unions to spend more than ever before to support or oppose particular candidates. Given that spending on political advertising has the potential to affect close election outcomes, my findings reinforce concerns that a small number of megadonors and special interests can use their wealth to distort the democratic process.

The rest of the paper proceeds as follows. Section 2 reviews related literature. Section 3 presents the data. Section 4 documents several key correlations in the data related to candidates' advertising strategies and the polls. Section 5 lays out the model and estimation strategy. Section 6 presents the empirical findings. Section 7 studies the impacts of counterfactual public financing and negativity blackout policies. Section 8 concludes.

2 Related Literature

This paper is related to several literatures. Most closely related is work studying the effects of political advertising on voter behavior. Since the advent of the so-called minimal effects hypothesis (Lazarsfeld et al., 1944; Klapper, 1960), scholars have doubted whether campaigning has any meaningful effect on voters' decisions. Modern field experiments have reinforced these doubts, arguing that the persuasive effects of political advertising are extremely short-lived (Gerber et al., 2011), practically zero (Kalla and Broockman, 2018), and are not masking larger heterogeneous effects that cancel out in the aggregate (Coppock et al., 2020). Recent work on commercial advertising for consumer packaged goods (Shapiro et al., 2021) and internet search advertising (Blake et al., 2015) echo these minimal effects.

In contrast, a recent strand of observational studies has relied on the misalignment of media market and state boundaries or instrumental variables as sources of plausibly exogenous variation in advertising exposure. They find positive and economically significant effects of advertising on candidates' vote shares (Gordon and Hartmann, 2013; Spenkuch and Toniatti, 2018), that these effects are even larger in down ballot races (Sides et al., 2021), and that positive and negative ads exhibit asymmetric effects on voter demand (Gordon et al., 2022).[2]

Complementing these reduced form and experimental studies, I extend the literature by building and estimating a dynamic equilibrium model of political

[2]These larger advertising effects are in line with studies that exploit natural experiments to identify the effects of advertising in consolidating democracies (Da Silveira and De Mello, 2011; Larreguy et al., 2018).

advertising competition.[3] By explicitly modeling candidates' advertising decisions, such an approach allows me to account for candidates' strategic interactions and forward-looking behavior that cannot be captured with reduced-form or experimental methods. Since the goal of this paper is to study the effect political advertising has on actual election outcomes, as opposed to voters' reported vote intentions in surveys or experimental settings, ignoring either the strategic responses or the advertising dynamics I document in the data could severely bias the estimates of advertising's effect on who wins the election. A structural approach also allows me to conduct novel counterfactual policy experiments to quantify the significance of unequal fundraising and negative advertising in Senate elections.

This paper contributes more broadly to the literature using dynamic advertising models to study how industry structure affects firms' optimal advertising strategies (e.g., Chintagunta and Vilcassim (1992) and Vilcassim (1999)). Most similar to this paper are Dubé et al. (2005) and Doraszelski and Markovich (2006), who use numerical methods to compute Markov Perfect Equilibria in dynamic discrete oligopoly models of product market advertising.

By modeling dynamic advertising competition between candidates, this paper relates to the literature estimating dynamic oligopoly models to study the role of market structure on equilibrium outcomes. These empirical dynamic games have been applied to such diverse contexts as hospital markets (Gowrisankaran and Town, 1997), the commercial aircraft industry (Benkard, 2004), innovation in PC microprocessors (Goettler and Gordon, 2011), and environmental regulation (Ryan, 2012).[4] Most related to this paper is Igami's (2017) study of the hard disk drive industry. He shows how modeling assumptions can accommodate the estimation of non-stationary models by ensuring a unique equilibrium and

[3]For other examples of structural models of political advertising, see Lovett and Shachar (2011), who estimate a static model of positive and negative advertising to explain why U.S. House candidates in more competitive elections run more negative campaigns, and Gordon and Hartmann (2016), who estimate a static model of where presidential candidates target their advertising.

[4]See Aguirregabiria et al. (2021) for recent comprehensive overview of the literature.

computational tractability.

Another pioneering paper is Iaryczower et al. (2022). They estimate a single-agent dynamic model of an incumbent running for reelection who can moderate his policy position and spend on advertising to improve his standings in the polls. They use their estimated model to quantify how electoral vulnerability affects incumbents' accountability. A key innovation of their paper is to exploit within-cycle variation in polls, position-taking and advertising for identification, a strategy which I adopt in this paper. Our papers differ in that my model features competing candidates whose ideological platform is assumed exogenously fixed from the beginning of the election.[5] I instead focus on how candidates' ad spending and negativity decisions influence election outcomes.

At a broader level, this paper adds to a recent literature in political economy using theory-based estimation of election models to evaluate hypothetical institutional reforms. Examples include Diermeier et al. (2005), who specify a dynamic model of career decisions of a member of Congress and study the effects of term limits and Congressional wage hikes. Knight and Schiff (2010) estimate a model of social learning in U.S. presidential primaries and study the consequences of eliminating staggered primaries. Sieg and Yoon (2017, 2022) estimate dynamic games of electoral competition with asymmetric information and evaluate the impact of term limits. My model and focus on candidates' within-election advertising strategies differs significantly from all those papers.

3 Data

I focus my attention on Senate candidates' general election advertising on local broadcast television between 2000 and 2018. Unlike U.S. House races, general

[5]Iaryczower et al. study candidates' decisions over several months leading up to election day, over which candidates may more effectively adjust their policy platforms. As I explain below, I focus on a shorter ten week period before election day when candidates' platforms have likely already been set.

elections to the Senate are high-profile, statewide races where candidates rarely run unopposed and where high frequency polling data is available.

Local broadcast television has traditionally been candidates' primary marketing tool for influencing voter support because of its broad reach and scalability. Of the estimated 2.6 billion dollars spent on political advertising in the 2008 general elections, the midpoint of my sample, approximately 2 billion dollars were spent on local broadcast television (Seelye, 2008). While digital advertising has grown as a fraction of all political ad spending, it still made up only 1 percent of all ad spending in 2014 (Fowler et al., 2020).

Broadcast television advertising data comes from Kantar Research's Campaign Media Analysis Group (CMAG) in conjunction with the Wesleyan Media Project (WMP). Since 2000, CMAG has used automated methods to track each time a campaign ad is aired over the course of an election. Subscribers to CMAG, including the political campaigns themselves, receive email alerts when a new ad is aired for the first time. As a result, campaigns are able to monitor the advertising strategies of their opponents in real time. Following each election, WMP's coders label each ad as either positive or negative. Positive ads are those that only mention the favored candidate, whereas negative ads explicitly mention the opponent.[6]

In addition to coding each ad's tone, CMAG includes estimated costs for each ad airing. Following standard practice, I convert these estimated expenditures into gross rating points (GRPs). One GRP corresponds to one percent of a market's television households and should be interpreted as the average number of times a typical household has been exposed to an ad over a given time period. GRPs are preferred to ad expenditures because they account for variation in the value, or exposure rates, of a 30 second spot across markets and more accurately depict the impact of advertising on voters' choice behavior.

I compile polling data from *Polling Report*, *Real Clear Politics*, and *Pollster*.

[6] See Appendix A for more details on the coding of positive and negative ads.

These sources record polls that were posted publicly in real time during the campaigns and were available to candidates when they made their budget and ad tone decisions. The pollsters poll likely voters and report the share of voters who say they will vote for each candidate.

I supplement the two main data sources above with election returns from the MIT Election Lab along with state economic conditions and demographics from FRED and the U.S. Census. I also include candidates' estimated ideologies from the DIME database (Bonica, 2013),[7] as well as indicators for whether a race was predicted to be a toss up before the election began from Cook's Political Report.[8] Additional data description is in Appendix A.

My final sample consists of 170 elections. Since Labor Day has traditionally marked the unofficial kickoff of general election season, I focus on the ten weeks leading up to election day and discretize these weeks into two-week-long periods.[9] I use this time period discretization in both the descriptive analysis of the data and in estimation.

I follow standard practice in the literature and aggregate all ad spending on behalf of a candidate, whether by the official campaign, the party, or political action committees (PACs), into a single spending measure. This measure has the advantage of capturing the broader information environment available to vot-

[7] These campaign donation-based measures of ideology have been extensively validated across several studies spanning a variety of institutional settings and types of actors (e.g., Bonica, 2018; Bonica, 2019). Other studies have also relied upon donation-based measures of candidate ideology to study extremist candidate behavior (e.g., Hall (2015, 2017)).

[8] Cook's Political Report rates the predicted competitiveness of races throughout the election year. Toss ups are expected to be the most competitive races whose outcome is the least predictable. While they classify non-toss up races more finely into other categories such as "lean Democrat" or "safe Republican", only the toss up classification is available for the first year in my sample. I use their classification of races from early August, before the general election begins, to identify races as toss ups.

[9] For a two-week period, I compute the polling margin as the weighted average of all polling margins in that period where the weights are proportional to the number of potential voters polled.

ers throughout the campaign, but it assumes that all advertising on behalf of a candidate is coordinated.

While such coordination technically violates campaign finance regulations, candidates frequently exploit loopholes or ignore the restrictions altogether to coordinate with PACs and party committees (Lee et al., 2014; Roarty and Goldmacher, 2014). Even the chairwoman of the Federal Election Commission, which is tasked with enforcing such regulations, admitted in 2016 that, "the likelihood of the laws being enforced is slim" (Lichtblau, 2015). As such, in both the analysis of the data and the structural estimation, I treat the official campaign and its supporting party and PACs as a single decision maker.

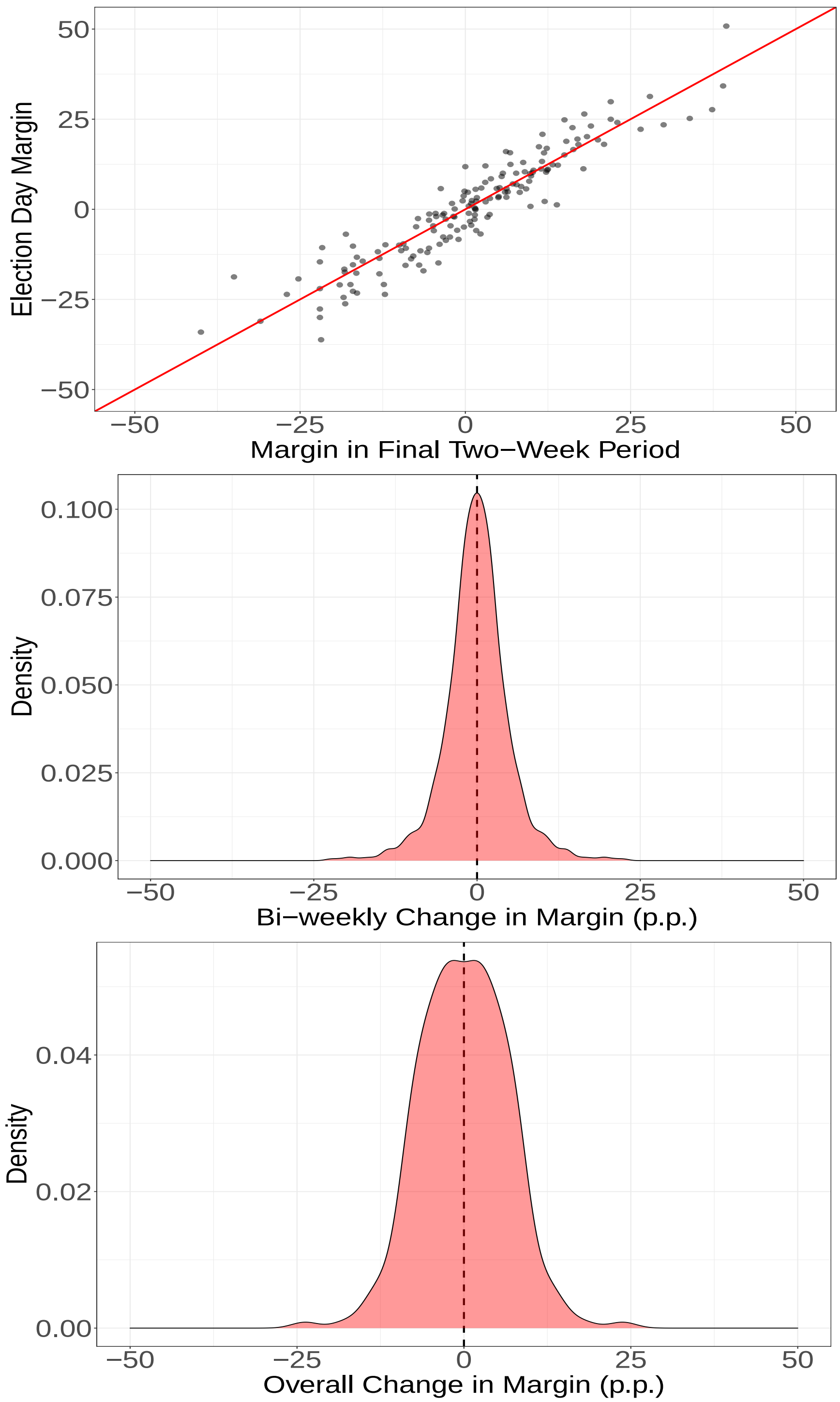
4 Descriptive Facts

The data reveal some new insights into the nature of political advertising and the role of strategic competition. In this section, I document the main empirical regularities that a model of campaign advertising needs to capture.

Figure 1 shows three facts about the polling margins. First, the polls are predictive of the election outcomes, so improving a candidate's standings in the polls makes them more likely to win on election day. Second, the polls are persistent across two-week-long time periods, which explains why candidates begin spending weeks before election day. Third, there is large amounts of variation in the polls from the first two-weeks of the election to the final election day result.

Elections separate naturally into those seats that are considered likely to be close before the election begins ("toss ups") and those that are considered safe. Close elections receive greater attention from parties, the media, and voters than safe seats. As their name suggests, toss up races begin with substantially closer polling margins than non-toss up races. While incumbents generally enjoy a large initial lead in the polls, their average initial polling advantage in toss up races is only around one percentage point.

Figure 1: Variation in the Polling Margins



Note: The top panel plots the difference in the polling margin in the final two-week-long period against the relative vote share on election day. The center panel plots the distribution of the period-to-period changes in the polling margin. The bottom panel plots the distribution of changes in the margin from the first two-week period of the election to the last.

Table 1 shows that spending and negativity are heavily concentrated in toss up races. There are, however, important asymmetries across different types of candidates. Incumbents in safe seats run overwhelmingly positive campaigns compared to incumbents in toss ups, while challengers and open seat candidates run majority negative campaigns regardless. Challengers are more ideologically extreme than the incumbents they face, and this is especially true for toss ups.

Just as spending and negativity are not evenly distributed among toss ups and safe seats, neither are they distributed evenly throughout the election. Figure 2 plots the average spending on positive and negative ads in each two-week period of the election. It shows that spending and negativity both increase as election day nears, and this is true for all types of candidates.

To get an idea of what drives candidates to first go negative, I estimate a Cox proportional hazard rate model where an event is a candidate airing her first negative ad. The estimates in Table 2 suggest candidates respond to one another and speed up their time to going negative once their opponent has gone negative. Candidates go negative earlier when the race is closer and when they are doing worse in the race. Candidates who are more ideologically extreme or who face more ideologically extreme opponents also go negative earlier.

To better understand what influences candidates' decisions over the course of the campaign, I regress a candidate's current period spending and negativity on their standing in the polls and their opponent's lagged advertising decision. Table 3 shows that, from period to period, candidates' spending and negativity respond to how they are doing in the polls, as well as their opponents' actions. Opposing candidates' spending and negativity are positively correlated, and candidates spend more and are more negative when the race is closer and when they are behind in the polls.

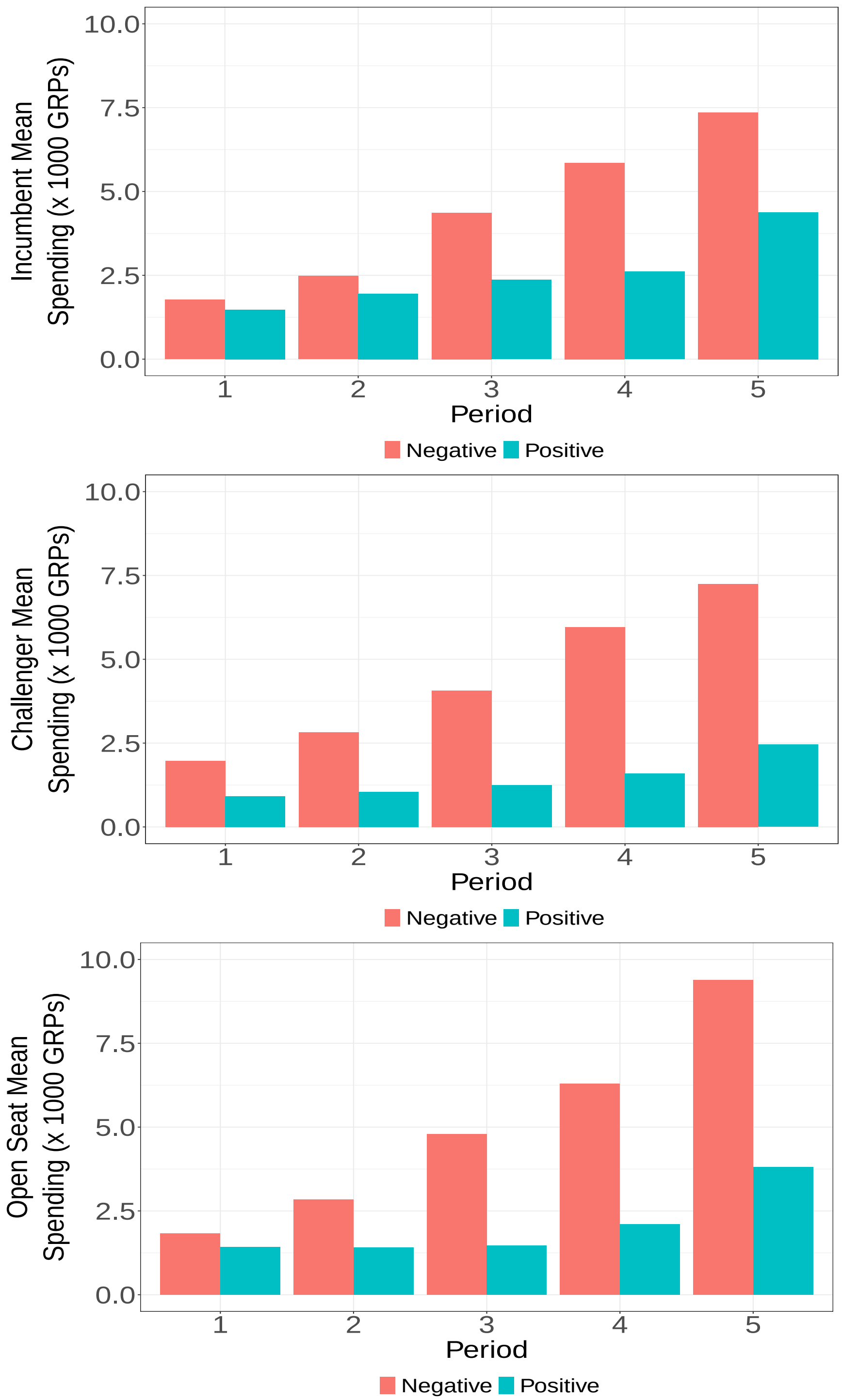
Taken together, the data paint a clear picture of candidate competition. Candidates spend more and go more negative when the race is close and as election day nears. Their advertising decisions depend on how they are doing in the polls,

Table 1: Summary Statistics by Incumbency and Toss Up Status

	Number of Candidates	Percent Democrat	Mean Initial Polling Advantage	Mean Percent Negative Ads	Mean Ad Spending ($\times 1,000$ GRPs)	Mean Democrat Ideology	Mean Republican Ideology
Safe Incumbent	77	54.55	12.70	36.94	25.14	-0.90	0.83
			(11.92)	(29.85)	(24.05)	(0.28)	(0.28)
Toss Up Incumbent	38	39.47	1.10	70.18	51.81	-0.91	0.84
			(4.12)	(20.21)	(32.09)	(0.32)	(0.27)
Safe Challenger	77	45.45	-12.70	63.28	18.19	-0.98	0.99
			(11.92)	(28.40)	(25.74)	(0.34)	(0.33)
Toss Up Challenger	38	60.53	-1.10	71.83	52.09	-1.10	1.18
			(4.31)	(17.84)	(41.01)	(0.26)	(0.17)
Open Seat	110	50.00	0.00	63.27	35.38	-1.01	1.02
			(17.32)	(19.85)	(16.38)	(0.33)	(0.21)
Total	340						

Note: Table shows summary statistics for candidates broken down by incumbency status and by whether or not the race was considered a toss up by Cook's Political Report in August. Ideology scores come from the DIME database and are constrained to a one-dimensional scale of -2 for the most liberal candidates and 2 for the most conservative. Standard deviations are in parentheses.

Figure 2: Ad Spending Over the Election



Note: From top to bottom, the panels show the mean spending on positive and negative ads by incumbents, challengers, and open seat candidates, respectively, in each two-week period of the election. Period one is the first period of the election. Period five ends on election day.

Table 2: What Drives Candidates to Go Negative Earlier?

	Cox Proportional Hazards Model:
	Time to First Negative Ad
Dummy: Opponent Went Negative	0.559^{***}
	(0.170)
Polling Advantage	-0.013^{**}
	(0.007)
|Polling Advantage|	-0.074^{***}
	(0.010)
Dummy: Incumbent	-0.326^{*}
	(0.190)
Dummy: Open Seat	-0.249
	(0.181)
|Ideology|	0.421^{*}
	(0.250)
|Opponent Ideology|	0.521^{**}
	(0.218)
Dummy: Toss Up	0.873^{***}
	(0.164)
Observations	608
Number of Events	313

Note: The sample consists of all candidate-periods leading up to and including the period a candidate first goes negative. An event is a candidate-period in which the candidate airs his first negative ad. A positive (negative) coefficient means an increase in the covariate is associated with a higher (lower) chance of a candidate going negative at any point in time. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 3: What Explains Negativity and Spending Choices?

	Dependent variable:	
	Negativity_t	Spending_t ($\times 1,000$ GRPs)
$\text{Polling Advantage}_t$	-0.276^{***}	-0.062^{**}
	(0.090)	(0.051)
$\lvert\text{Polling Advantage}\rvert_t$	-0.807^{***}	-0.236^{**}
	(0.180)	(0.049)
$\text{Opponent Negativity}_{t-1}$	0.110^{***}	0.007^{*}
	(0.029)	(0.004)
$\text{Opponent Spending}_{t-1}$	2.220^{***}	0.651^{***}
	(0.134)	(0.072)
|Ideology|	4.240^{***}	1.201
	(1.295)	(0.783)
|Opponent Ideology|	3.735^{***}	1.376^{**}
	(1.103)	(0.611)
Dummy: Incumbent	-2.307	-0.032
	(2.310)	(0.397)
Dummy: Open Seat	0.932	0.128
	(1.871)	(0.413)
Controls	Y	Y
Year FE	Y	Y
Time Period FE	Y	Y
Observations	1,040	1,040
Adjusted R^2	0.431	0.713

Note: Standard errors are clustered at the year-state level. Negativity is measured in percentage points between 0 and 100. State-level controls include real median household income, percent 65+, percent black, percent Hispanic, percent urban, percent unemployed, leading economic indicator, the most recent presidential Republican vote share, and a dummy variable for whether the race was considered a toss up by Cook's Political Report at the beginning of the election. Candidate controls include party and candidate gender. *p<0.1; **p<0.05; ***p<0.01

as well as on their opponents' choices. Candidates' ideologies also play a role, with more extreme candidates both running more negative campaigns and being targeted by more negative campaigns than their moderate counterparts.

5 Empirical Framework

I model an election between two candidates as a non-stationary dynamic discrete game with incomplete information similar to that of Aguirregabiria and Mira (2007) and Igami (2017). Time is discrete with a finite horizon, $t = 0, 1, ..., T$, where period T is election day. I define the candidate $i \in \{C, I\}$ whose party most recently held the seat as the incumbent I and the opponent as the challenger C.

5.1 Control and State Variables

In every period $t = 0, ..., T-1$ leading up to election day, each candidate i chooses among $j = 1, 2, ..., J$ mutually-exclusive discrete alternatives. The jth alternative consists of a level of advertising expenditures e_j and, if $e_j > 0$, a fraction n_j of those expenditures devoted to negative ads. Let $a_{ijt} \in \{0, 1\}$ be an indicator equal to one if candidate i chose alternative j at time t, and zero otherwise. Let the action vector $a_{it} = (a_{i1t}, ..., a_{iJt}) \in \mathcal{A}$ characterize candidate i's choices at time t. Because alternatives are mutually exclusive,

$$\sum_{j=1}^{J} a_{ijt} = 1 \tag{1}$$

Let e_{it} and n_{it} denote the spending and negativity choices associated a_{it}:

$$e_{it} = \sum_{j=1}^{J} a_{ijt} e_j \quad \text{and} \quad n_{it} = \sum_{j=1}^{J} a_{ijt} n_j \tag{2}$$

Let $a_t = (a_{Ct}, a_{It})$ denote the period t action profile.

At the start of period t, a candidate is characterized by two vectors of state variables that affect his payoff, x_t and ε_{it}. The observable K-dimensional vector $x_t \in X$ contains variables that are common knowledge for both candidates, but the unobservable J-length vector $\varepsilon_{it} = (\varepsilon_{i1t}, ..., \varepsilon_{iJt}) \in \mathcal{E}_i$ contains alternative-specific random shocks that are candidate i's private information.[10] Vector $x_t = (p_t, y_t)$ consists of one stochastically evolving state variable, the incumbent's relative polling margin p_t, as well as a vector y_t of deterministic candidate- and election-specific variables detailed below. Moreover, x_t is assumed to only take on finitely many values.

Candidates have beliefs over uncertain future states of the world. These beliefs are described by a Markov transition density of the state variables, $p(x_{t+1}, \varepsilon_{t+1} | x_t, \varepsilon_t, a_t)$, where $\varepsilon_t = (\varepsilon_{Ct}, \varepsilon_{It})$. I assume this density satisfies the following conditional independence assumption:

$$p(x_{t+1}, \varepsilon_{t+1} | x_t, \varepsilon_t, a_t) = g(\varepsilon_{t+1})\; f(x_{t+1} | x_t, a_t) \tag{3}$$

Equation (3) shows that, conditional on candidates' actions, the private information state variables do not influence the evolution of the commonly observed state variables. Moreover, private information variables are assumed i.i.d. across actions, candidates, and time periods, but candidates know their density.

5.2 Period Utility

Candidate i's period utility depends on the commonly known state x_t, on his own private information ε_{it}, and his current action choice a_{it}. Private information appears additively in candidates' period utility function. In each period $t = 0, ..., T-1$ leading up to election day, candidate i's period utility is given by

[10] These private shocks may reflect, among other things, idiosyncratic informational and managerial differences in campaigns, such as private opposition research and opinion polling, feedback from focus groups, unforeseen donations, or differences in the advising of campaign strategists.

$$u_{it}(a_{it}, x_t, \varepsilon_{it}) = \sum_{j=1}^{J} a_{ijt}[-c_{ij}(x_t) + \varepsilon_{ijt}] \tag{4}$$

where $c_{ij}(x_t)$ is candidate i's opportunity cost of spending e_j on ads in period t when the state is x_t. These opportunity costs reflect the depths of candidates' war chests as well as their fundraising abilities. They are common knowledge, consistent with mandatory financial disclosure laws for campaign donations and spending.[11]

On election day in period T, candidate i's period utility is given by

$$u_{iT}(p_T) = I\{\text{win}_i(p_T)\}\ \omega \tag{5}$$

where $I\{\text{win}_i(p_T)\}$ is an indicator function for whether candidate i's relative polling share is greater than zero in state p_T. The payoff ω for winning the race reflects the benefits to a Senator from holding office.

5.3 Timing

I follow Igami (2017) and assume that candidates move sequentially within each period, with the incumbent choosing his action before the challenger. The payoff from such an assumption is twofold. First, when the model is finite horizon and players move sequentially, there is a unique equilibrium as long as there is a single utility-maximizing choice among the set of discrete alternatives for each candidate in each period. My functional form assumptions, combined with the fact that the

[11]This formulation of candidates' advertising costs abstracts from candidates' intertemporal spending and savings decisions. By modeling candidates' cost of advertising as a period opportunity cost, candidates' current funds are not additional state variables and candidates' savings decisions are not additional choice variables. While intertemporal savings decisions are potentially interesting in their own right, the focus of this paper is how advertising influences election results. Abstracting from these savings decisions allows me to keep tractable a richer model of advertising competition.

unobserved state variables are draws from a continuous distribution, ensure that this is the case.

Second, solving the sequential-move game is much less computationally demanding than solving the simultaneous-move game, which may give rise to multiple equilibria. When it is a candidate's turn to move, she is essentially solving a single-agent problem given her beliefs about her opponent's behavior and the evolution of the state variables. Moreover, realizations of the unobserved state variables only impact future payoffs through the actions chosen. Candidates thus have perfect information over the payoff relevant history. As a result, solving the sequential-move game only requires solving a sequence of single-agent problems by backward induction, whereas solving the simultaneous-move game requires computing a fixed point in the equilibrium choice probabilities in each period.

5.4 Bellman Equations and Equilibrium

A strategy for the incumbent is a set of mappings $\sigma_I = \{\sigma_{It} : t = 0, ..., T-1\}$ from the state variables he observes into actions, $\sigma_{It} : X \times \mathcal{E}_I \to \mathcal{A}$. Because the challenger observes the incumbent's current action before choosing her alternative, a strategy for the challenger is a set of mappings $\sigma_C = \{\sigma_{Ct} : t = 0, ..., T-1\}$ where $\sigma_{Ct} : X \times \mathcal{E}_C \times \mathcal{A} \to \mathcal{A}$. Let $\sigma = (\sigma_C, \sigma_I)$ be a strategy profile.

Associated with a strategy profile σ is a set of conditional choice probabilities $P^\sigma = \{P^\sigma_{It}(a_{It}|x_t), P^\sigma_{Ct}(a_{Ct}|x_t, a_{It}) : t = 0, ..., T-1\}$ where

$$\begin{aligned} P^\sigma_{It}(a_{It}|x_t) &= \Pr(\sigma_{It}(x_t, \varepsilon_{It}) = a_{It}|x_t) \\ &= \int I\{\sigma_{It}(x_t, \varepsilon_{It}) = a_{It}\} g(\varepsilon_{It}) d\varepsilon_{It} \end{aligned} \tag{6}$$

$$\begin{aligned} P^\sigma_{Ct}(a_{Ct}|x_t, a_{It}) &= \Pr(\sigma_{Ct}(x_t, \varepsilon_{Ct}, a_{It}) = a_{Ct}|x_t, a_{It}) \\ &= \int I\{\sigma_{Ct}(x_t, \varepsilon_{Ct}, a_{It}) = a_{Ct}\} g(\varepsilon_{Ct}) d\varepsilon_{Ct} \end{aligned} \tag{7}$$

and where $I\{\cdot\}$ is the indicator function. Because candidates do not observe each others' private information shocks, these choice probabilities constitute a candidate's beliefs about his opponent's behavior when his opponent plays according to her strategy in σ.

Candidates are rational and forward-looking. They account for the optimal dynamic behavior of their opponents when choosing their ad spending and negativity to maximize their expected future utility. When it is a candidate's turn to move, he is effectively solving a single-agent dynamic programming problem given his beliefs about his opponent's behavior and the evolution of the polls. The solution to candidates' dynamic programs is a set of period value functions $\{V_{i0}, ..., V_{iT}\}$ satisfying the Bellman equations:

$$V^{\sigma}_{It}(x_t, \varepsilon_{It}) = \max_{a_{It} \in \mathcal{A}} \Bigg\{ u_{It}(a_{It}, x_t, \varepsilon_{It}) + \sum_{x_{t+1} \in X} \sum_{a_{Ct} \in \mathcal{A}} \int V^{\sigma}_{It+1}(x_{t+1}, \varepsilon_{It+1}) dG(\varepsilon_{It+1}) P^{\sigma}_{Ct}(a_{Ct}|x_t, a_{It}) f(x_{t+1}|x_t, a_t) \Bigg\} \tag{8}$$

$$V^{\sigma}_{Ct}(x_t, \varepsilon_{Ct}, a_{It}) = \max_{a_{Ct} \in \mathcal{A}} \Bigg\{ u_{Ct}(a_{Ct}, x_t, \varepsilon_{Ct}) + \sum_{x_{t+1} \in X} \sum_{a_{It+1} \in \mathcal{A}} \int V^{\sigma}_{Ct+1}(x_{t+1}, \varepsilon_{Ct+1}, a_{It+1}) dG(\varepsilon_{Ct+1}) P^{\sigma}_{It+1}(a_{It+1}|x_{t+1}) f(x_{t+1}|x_t, a_t) \Bigg\} \tag{9}$$

where the terminal value functions are $V_{iT}(p_T) = u_{iT}(p_T)$.

A strategy profile σ is a *Markov Perfect Equilibrium (MPE)* if for each candidate i, σ_i is a best response to σ_{-i}. The main assumption behind MPE is that both candidates' strategies in period t are functions only of the period t payoff-relevant state variables. As discussed in the previous section, the finite time horizon and sequential moves assumption guarantee that there is a unique MPE of the game that I denote by σ below.

5.5 Empirical Specification

In addition to the polling margin p_t, the state space consists of a vector y_t containing several commonly observed variables capturing candidate- and election-specific characteristics with deterministic transitions. These include candidates' ideological extremeness, an indicator for whether the race was considered a toss up, the state's unemployment rate[12] and partisanship index,[13] and a post-*Citizens United* indicator variable.[14] I also include indicators equal to one if a candidate has already starting airing negative ads by the start of period t and zero otherwise. With the exception of the indicators for whether candidates have gone negative, these variables are fixed from the start of the election.

I parameterize the period opportunity cost of spending an amount e_{it} on advertising by

$$c_{ij}(x_t) = \exp(y_{1t}'\kappa_i)e_j^2 \tag{10}$$

where $y_{1t} \subset y_t$ consists of a constant, an indicator for whether the race is for an open seat, an indicator for whether the race was considered a toss up by Cook's Political Report by the start of August, and an indicator for whether the race took place after *Citizens United*.

The main modeling decision I must make is to determine how the polls evolve as a function of candidates' advertising. I assume the following transition equation:

[12] A district's unemployment rate is commonly used to proxy for local economic conditions.

[13] I follow the literature and use the state's support for the candidate's party in the previous presidential election as a proxy for the state's partisan leaning.

[14] Rulings in *Citizens United* and *SpeechNow v. FEC* were decided within weeks of each other. In the discussion that follows, I refer to the collective impact of both *Citizens United* and *SpeechNow v. FEC* as just *Citizens United*.

$$p_{t+1} = \rho p_t + \underbrace{\beta_{I1}(e_{It}n_{It})^{\tau_{I1}} - \beta_{C1}(e_{Ct}n_{Ct})^{\tau_{C1}}}_{\text{Difference in Negative Spending}} +$$

$$\underbrace{\beta_{I2}(e_{It}(1-n_{It}))^{\tau_{I2}} - \beta_{C2}(e_{Ct}(1-n_{Ct}))^{\tau_{C2}}}_{\text{Difference in Positive Spending}} + \tag{11}$$

$$y_{2t}'\phi + \nu_t$$

where $y_{2t} \subset y_t$ contains an intercept, the state unemployment rate, and the state partisanship index, where $0 < \tau_{i1}, \tau_{i2} < 1$, and where ν_t is drawn i.i.d. from $\mathcal{N}(0, \sigma_\nu^2)$ after candidates have chosen their period t actions.[15]

There are several important features of the specification in equation (11). First, dynamics in the model stem from the persistence in the polling equation. Forward-looking candidates seeking to improve their standings in the polls are driven to advertise well in advance of election day because only a fraction of their contemporaneous advertising effects will carry over into future periods.

Second, separate slope and concavity parameters on positive and negative spending allow for the possibility that negative information is more persuasive than positive information and can account for asymmetric ad tone effects. Letting the slope coefficients further depend on incumbency status allows for the chance that some candidates may be better at airing one type of ad than others. Incumbents, for example, may have an advantage at airing positive ads because they can highlight their accomplishments from previous terms. However, they may be worse at airing negative ads against a challenger who has never held the

[15] An advantage of specifying a reduced form for how advertising translates into voter support is that I avoid taking a stance on the exact mechanism through which positive and negative advertising influences voters. While the theoretical literature has proposed various such mechanisms (Baron, 1994; Skaperdas and Grofman, 1995; Harrington and Hess, 1996; Prat 2002; Ashworth, 2006; Polborn and Yi, 2006; Hao and Li, 2013; and Bostanci et al., 2022), there is no consensus from the empirical literature on the importance of any one mechanism.

office and has less of a political record to criticize. Challengers, by contrast, have an incumbent's record to criticize, but less of a record of their own to promote.

Finally, inclusion of the error term, ν_t, accounts for the possibility that some aspects of advertising, like idiosyncrasies in ad content or variation in the composition of viewers, are not captured by the data.

In order for the model to explain the heterogeneous levels of negativity in the data, I allow for the effectiveness of positive and negative spending to vary with observables in estimation. Specifically, I assume that

$$\beta_{i1} = \beta_{i10} + \beta_{i11}\ (\text{Gone Negative}_{-it} \times \text{Ideology}_{-i})\ + \beta_{i12}\ \text{Toss Up} + \beta_{i13}\ \text{Open Seat}$$
$$\beta_{i2} = \beta_{i20} + \beta_{i21}\ \text{Open Seat} + \beta_{i22}\ \text{Ideology}_i$$

Under this specification, the effectiveness of a candidates' negative ads depends on the characteristics of her opponent, whereas the effectiveness of a candidates' positive ads depends on her own characteristics.

5.6 Discussion of Model Mechanisms

Equations (8) and (9) illustrate candidates' main dynamic trade off. Candidates trade off the cost of advertising today with the marginal return of improving their standings in the polls next period, thereby increasing the probability they eventually win the race. This trade off drives candidates to spend more when the race is closer. Moreover, persistence in the polling transition equation generates a higher marginal return to advertising closer to election day. This persistence explains why candidates spend more at the end of the race than at the beginning.

Because I allow for positive and negative advertising to exhibit different rates of diminishing marginal returns, the model can generate correlation between spending and negativity. Specifically, if $\tau_{i1} > \tau_{i2}$, candidate i will spend more on negative ads as he increases his total spending, which happens both when the race is closer and as election day nears.

The model can also account for the timing decisions of when candidates go negative through the inclusion of the indicator state variables for whether the opponent has already gone negative in the returns to negative spending. By interacting this dummy with the opponent's ideology, candidates who face more ideologically extreme candidates may be driven to go negative earlier than those facing moderates.

Finally, controlling for variation in candidate characteristics and local political conditions in both candidates' opportunity costs and the transition equation allows the model to flexibly capture the heterogeneity in candidate strategies in the data.

5.7 Identification and Estimation

Intuitively, identification of the transition equation parameters comes from the responsiveness of the polls to candidates' positive and negative advertising as they optimally respond to unobserved idiosyncratic shocks over the course of the election. Because candidates trade off their cost of advertising today with the marginal return of improving the likelihood they eventually win the race, observing how candidates' spending decisions vary with their standings in the polls allows me to recover their opportunity cost parameters. The payoff parameter then pins down the levels of ad spending.

To keep the estimation tractable, I follow Rust (1987) and assume that the ε_{it} are distributed i.i.d. multivariate Type-I Extreme Value with location zero and scale one. Exploiting the properties of the extreme value distribution and integrating over the private information shocks in (8) and (9) gives candidates' integrated Bellman equations

$$\widetilde{V}^{\sigma}_{It}(x_t) = \log\left\{\sum_{a_{It}\in\mathcal{A}} \exp\left[v^{\sigma}_{It}(a_{It}|x_t)\right]\right\} \tag{12}$$

$$\widetilde{V}^{\sigma}_{Ct}(x_t, a_{It}) = \log\left\{\sum_{a_{Ct}\in\mathcal{A}} \exp\left[v^{\sigma}_{Ct}(a_{Ct}|x_t, a_{It})\right]\right\} \tag{13}$$

where $\widetilde{V}^{\sigma}_{It}(x_t) = \int V^{\sigma}_{It}(x_t, \varepsilon_{It})dG(\varepsilon_{It})$ and $\widetilde{V}^{\sigma}_{Ct}(x_t, a_{It}) = \int V^{\sigma}_{Ct}(x_t, \varepsilon_{Ct}, a_{It})dG(\varepsilon_{Ct})$ are candidates' integrated value functions, and where candidates' choice-specific value functions are given by

$$\begin{aligned} v^{\sigma}_{It}(a_{It}|x_t) = -\left[\sum_{j=1}^{J} a_{Ijt}\ c_{Ij}(x_t)\right] + \\ \sum_{x_{t+1}\in X}\sum_{a_{Ct}\in\mathcal{A}} \widetilde{V}^{\sigma}_{It}(x_{t+1})P^{\sigma}_{C}(a_{Ct}|x_t, a_{It})f(x_{t+1}|x_t, a_t) \end{aligned} \tag{14}$$

$$\begin{aligned} v^{\sigma}_{Ct}(a_{Ct}|x_t, a_{It}) = -\left[\sum_{j=1}^{J} a_{Cjt}\ c_{Cj}(x_t)\right] + \\ \sum_{x_{t+1}\in X}\sum_{a_{It+1}\in\mathcal{A}} \widetilde{V}^{\sigma}_{Ct+1}(x_{t+1}, a_{It+1})P^{\sigma}_{I}(a_{It+1}|x_{t+1})f(x_{t+1}|x_t, a_t) \end{aligned} \tag{15}$$

The equilibrium conditional choice probabilities then take the familiar logit form:

$$P^{\sigma}_{It}(a_{It}|x_t) = \frac{\exp\{v^{\sigma}_{It}(a_{It}|x_t)\}}{\sum_{\widetilde{a}_{It}\in\mathcal{A}} \exp\{v^{\sigma}_{It}(\widetilde{a}_{It}|x_t)\}} \tag{16}$$

$$P^{\sigma}_{Ct}(a_{Ct}|x_t, a_{It}) = \frac{\exp\{v^{\sigma}_{Ct}(a_{Ct}|x_t, a_{It})\}}{\sum_{\widetilde{a}_{Ct}\in\mathcal{A}} \exp\{v^{\sigma}_{Ct}(\widetilde{a}_{Ct}|x_t, a_{It})\}} \tag{17}$$

I estimate the model on a panel $\{x_{mt}, a_{mt}\}$ of observed states and actions for $m = 1, ..., 170$ elections, each of which consists of $t = 0, 1, ..., 4$ two-week-long periods and an election day result at $t = 5$. I discretize the set of incumbent polling advantages into 20 equispaced bins between -20 and 50, the set of spending choices into 20 equispaced bins between 0 and 30,000 GRPs, and the negativity choices into 10 equispaced bins between 0 and 1. I discretize the polling transition equation using Tauchen's (1986) method to ensure that the set of polling margins is bounded and to obtain a discrete probability distribution over future polling margins. Appendix B provides additional details on the discretization and shows that the estimates are robust to using a finer level of discretization.

I estimate the model by full-information maximum likelihood using a nested fixed point algorithm as in Rust (1987). For a candidate parameter vector θ, I solve the model by backward induction to obtain the equilibrium choice probabilities. The choice probabilities and transition probabilities associated with the observed actions and states in the data form the basis for the likelihood function:

$$\prod_{m=1}^{M}\prod_{t=0}^{T-1} f(x_{mt+1}|x_{mt}, a_{mCt}, a_{mIt}; \theta) P_{It}^{\sigma}(a_{mIt}|x_{mt}; \theta) P_{Ct}^{\sigma}(a_{mCt}|x_{mt}, a_{mIt}; \theta) \tag{18}$$

The parameter space is then searched for the vector of parameters that best rationalizes the actual advertising choices and polling margins in the data.

6 Empirical Results

6.1 Parameter Estimates

Table 4 contains cost and payoff parameter estimates and asymptotic standard errors. I find that the benefits from holding office are significant and large in magnitude. In addition to their higher average polling margins, incumbents also face a substantially lower opportunity cost of advertising than challengers. On average, challengers have a 30 percent higher cost of fundraising than incumbents, and incumbents benefited more from the deregulation of campaign finance laws after *Citizens United*. Both candidates face lower costs to advertise in the more competitive toss up races, reflecting parties and interest groups channeling their resources to the races whose outcomes are least predictable from the start.

It is no surprise that incumbents face a lower opportunity cost of advertising than challengers. Six-year Senate terms give incumbents ample time in office to accumulate a war chest and expand their network of donors. Unlike challengers

Table 4: Preference Parameter Estimates and Asymptotic Standard Errors

Parameter	Description	Estimate	Standard Error
κ_{C0}	Challenger Cost Intercept	0.60	0.11
κ_{C1}	Toss Up	-0.43	0.07
κ_{C2}	Post-Citizens United	-0.33	0.08
κ_{C3}	Open Race	-0.11	0.06
κ_{I0}	Incumbent Cost Intercept	0.35	0.07
κ_{I1}	Toss Up	-0.27	0.05
κ_{I2}	Post-Citizens United	-0.44	0.11
κ_{I3}	Open Race	0.11	0.04
ω	Officeholder Benefits	3.10	0.81

Note: Table contains preference parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$).

from the non-incumbent party, incumbents rarely face a serious primary contender within their party that depletes their funds before the general election begins. Special interests seeking access to influential legislators and Congressional committees also tend to contribute to incumbents instead of challengers (Fouirnaies and Hall, 2014; Kalla and Broockman, 2016; Powell and Grimmer, 2016). This fundraising advantage, combined with incumbents' higher initial polling advantages, helps explain the high incumbent reelection rate in the data.

Transition parameters are in Table 5. The coefficients on positive and negative advertising have the expected signs and show that spending on both positive and negative ads are productive investments for both candidates. Negative advertising is more effective in toss up races than when the seat is safe. Challengers who are more ideologically extreme run less effective positive ads, though the analogous parameter estimate for incumbents is insignificant.

The returns to negative advertising increase when the opponent goes negative, and the increase in returns is positively related to the opponent's ideological extremism. As a result, candidates may strategically delay going negative in order to prevent increasing the effectiveness of their opponent's negative ads, especially when they are far ahead in the polls. However, because of the persistence in the polling equation, candidates who are doing poorly in the polls cannot afford to forego going negative for too long without sacrificing the chance to catch up before election day. These timing considerations explain why candidates who are lagging in the polls go negative sooner than those who are far ahead.

I normalize τ_{C2} and τ_{I2} to 0.5 in estimation so that τ_{C1} and τ_{I1} describe how much faster or slower the marginal returns to negative advertising decay relative to positive advertising. The fact that the estimates of τ_{C1} and τ_{I1} are both greater than 0.5 shows that negative ads feature a slower rate of diminishing marginal returns than positive ads. As a result, spending and negativity are positively correlated, and candidates devote more of their budget to negative advertising towards the end of the race and when the race is closer. That τ_{C1} is greater than τ_{I1} explains why the increase in challengers' negativity as they spend more money is steeper than for incumbents.

To compare my results with recent reduced-form findings in the literature on advertising in presidential races, I compute the average returns to positive and negative advertising for the candidates in my data. For each candidate in each period, I compute the contemporaneous increase in their relative polling share according to equation (11) when his equilibrium level of positive or negative ads is increased by one standard deviation, holding all else fixed. I then report the average increase in their relative share.

Increasing the average incumbent's positive ads by one standard deviation–

Table 5: Transition Parameter Estimates and Asymptotic Standard Errors

Parameter	Description	Estimate	Standard Error
β_{C10}	Negative Spending Intercept	1.13	0.09
β_{C11}	Incumbent Gone Negative × Incumbent Ideology	0.51	0.04
β_{C12}	Toss Up	0.22	0.03
β_{C13}	Open Seat	-0.09	0.05
β_{I10}	Negative Spending Intercept	0.39	0.03
β_{I11}	Challenger Gone Negative × Challenger Ideology	0.47	0.07
β_{I12}	Toss Up	0.07	0.03
β_{I13}	Open Seat	0.11	0.04
β_{C20}	Positive Spending Intercept	0.63	0.08
β_{C21}	Open Seat	0.03	0.02
β_{C22}	Challenger Ideology	-0.43	0.09
β_{I20}	Positive Spending Intercept	0.57	0.06
β_{I21}	Open Seat	-0.07	0.03
β_{I22}	Incumbent Ideology	-0.14	0.15
τ_{C1}	Challenger Power on Negative Spending	0.90	0.08
τ_{I1}	Incumbent Power on Negative Spending	0.67	0.07
ρ	Autoregressive Parameter	0.85	0.17
ϕ_0	Intercept	0.21	0.04
ϕ_1	Unemployment Rate	0.09	0.52
ϕ_2	Partisanship Index	0.48	0.21
σ_ν	Transition shock standard deviation	4.31	0.72

Note: Table contains transition parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$). The signs on the challenger's positive and negative spending parameters have been negated so that the estimates reflect the effect of advertising on the challenger's relative polling margin instead of the incumbent's.

equivalent to potential voters seeing an additional 60 more ads for the incumbent than for the challenger–leads to an immediate 0.46 percentage point increase in his relative polling advantage.[16] The average challenger faces a similar 0.42 percentage point return to positive advertising. Positive advertising thus benefits challengers and incumbents similarly on average.

In contrast, I find large asymmetries in the effectiveness of negative advertising. Increasing the incumbent's negative advertising by one standard deviation leads to an immediate 0.94 percentage point gain in his polling advantage average , while this gain is equal to 2.18 percentage points for the challenger.

Gordon et al. (2022) find that a one standard deviation increase in negative ads increases a candidate's relative vote share by 2.7 to 4.1 percentage points, while a one standard deviation increase in positive ads increases a candidate's relative vote share by 1 to 1.5 percentage points. Though they do not estimate separate positive and negative advertising effects, Spenkuch and Toniatti (2018) find that a one standard deviation shift in a presidential candidate's total advertising is associated with a shift in their relative vote margin of 0.49 to 0.67 percentage points. Sides et al. (2021) find a similar relative vote margin effect of 0.44 percentage points due to a one standard deviation change. While I study Senate elections instead of presidential elections, my estimates are similar in magnitude to those based on reduced-form methods.

Taken together, my findings show that political advertising is an effective marketing tool for all candidates. How this advertising influences electoral competition depends heavily on ad content and candidates' characteristics. Because the average return to positive advertising is similar for both incumbents and challengers, if candidates could only run positive ads, advertising would reinforce the incumbent's fundraising and initial polling advantages. In contrast, negative advertising is substantially more effective for challengers than for incumbents and works against incumbents' electoral advantages.

[16] One standard deviation in candidates' biweekly ad spending is equivalent to roughly 6,000 GRPs.

6.2 Robustness and Model Fit

The main concern with assuming a deterministic ordering of moves is that the ordering may generate an artificial first-mover advantage. In Appendix C, I re-estimate the model assuming the reverse ordering of moves and find that the parameter estimates are very similar regardless of the assumed order, suggesting that any artificial first-mover advantage in this context is minimal. In Appendix B, I re-estimate the model after increasing the number of bins used to discretize each variable and find that the results are virtually unchanged. In Appendix E, I compare moments related to candidates' advertising decisions and election outcomes in the real data with their counterparts in data simulated from the estimated model and show that the model can fit various features of the data quite well.

7 Policy Analysis

7.1 Counterfactual 1: Public Campaign Financing

The electoral advantage incumbents hold over non-incumbents is one of the best-documented patterns in U.S. elections, though there is much less agreement about its sources. This advantage is reflected in my estimates by incumbents' relatively lower opportunity cost to advertising. Given that advertising has the potential to improve candidates' standings in the polls, how much does this lower cost to advertise contribute to incumbents' electoral successes?

To limit the influence of wealth in politics and balance the spending of incumbents and challengers, many democracies provide public funds to candidates and limit their ability to raise money from private donors. Within the U.S., public campaign financing policies exist for presidential races and for various state and local races, but not for Congressional races.[17]

[17] Public financing options for Congressional candidates have been proposed in Congress. The

To quantify how much incumbents' fundraising advantage over challengers affects their reelection prospects, I use the estimated model to simulate a public financing policy that eliminates incumbents' fundraising advantage.[18] Specifically, I equalize the cost parameters for incumbents and challengers and generate 15,000 counterfactual elections with the same distribution of initial conditions as in the data.

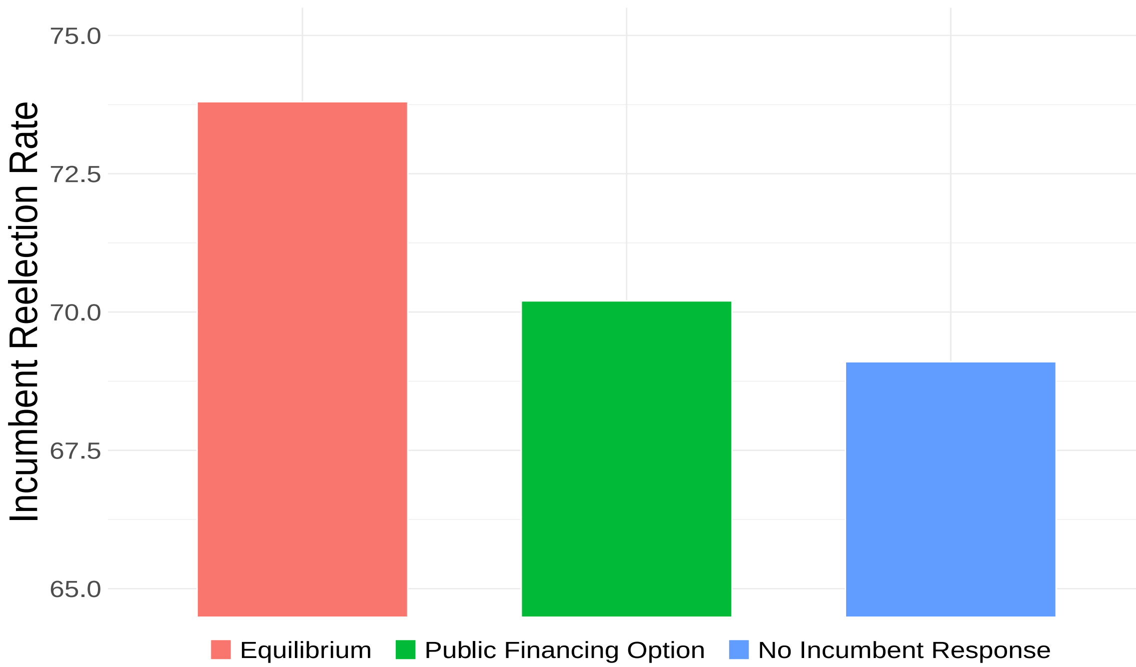
Figure 3 shows that eliminating the incumbent's fundraising advantage leads to a 3.6 percentage point drop in the incumbent reelection rate, from 73.8 percent to 70.2 percent. While this drop is sizeable on its own, the endogenous response by incumbents works against the policy's impact. With a lower cost to advertising, challengers spend more, causing the polling margins to narrow. But as the race becomes closer, the marginal return to advertising increases, and incumbents respond by spending more than when the race was less close.

To quantify the significance of incumbents' endogenous response to higher challenger spending and narrowing polls, I simulate the election outcomes when the incumbents' spending levels are set to their pre-public financing equilibrium levels while the challengers' spending remains at their post-public financing levels. Figure 3 shows that incumbents' strategic response eliminated nearly a quarter of the drop in the incumbent reelection rate caused by eliminating incumbents' fundraising advantage. Had incumbents not been able to respond, the incumbent reelection rate would have dropped by an even larger 4.7 percentage points. Ignoring strategic interactions would have substantially overstated the benefits of the policy, suggesting incumbents' fundraising advantage accounts for an even greater

Congressional Campaign Spending Limit and Election Reform Act of 1992 was passed by both the House and the Senate but ultimately vetoed by George H. W. Bush. Chief among Bush's reasons for vetoing the bill is that it would not, "reduce the unfair advantages of incumbency" (Bush, 1990).

[18]Because the model does not feature an intertemporal budget constraint that allows candidates to save money from period to period, the public financing policy being simulated most closely mimics a matching fund, where donations to one candidate are matched by a government-funded donation to the opposing candidate.

Figure 3: The Effect of Eliminating the Incumbents' Fundraising Advantage on the Incumbent Reelection Rate



Note: Figure shows the effect of eliminating the incumbent's fundraising advantage on the incumbent reelection rate. The left bar shows the baseline equilibrium incumbent reelection rate. The center bar shows the incumbent reelection rate under the public financing policy when their fundraising advantage is eliminated. The right bar shows the incumbent reelection rate under the public financing policy when the incumbents' spending is fixed at the baseline equilibrium levels.

fraction of their electoral success.

These findings relate to existing work on incumbent accountability, which has found that incumbents respond more to their constituents when their reelection prospects are in danger (Ansolabehere et al., 2001; Griffen, 2006; Mian et al., 2010; Iaryczower et al., 2022). This so-called marginality hypothesis argues that as incumbents become more electorally secure, they can shirk on exerting costly effort or implement their own preferred ideological platform instead of the policies preferred by their districts. High incumbent reelection rates and less competitive races, therefore, lead to worse representation for voters.

7.2 Counterfactual 2: Negativity Blackout

As my parameter estimates and public financing counterfactual show, candidates' advertising *can* make a difference in who wins close races, even when accounting for candidates' equilibrium responses. However, I also find large asymmetries in the effectiveness of positive and negative ads, suggesting that it is not just how much candidates spend that matters. How they divide their spending on positive and negative ads, and how these advertising decisions interact with both their own and their opponent's political backgrounds and ideological platforms may also impact who wins the election.

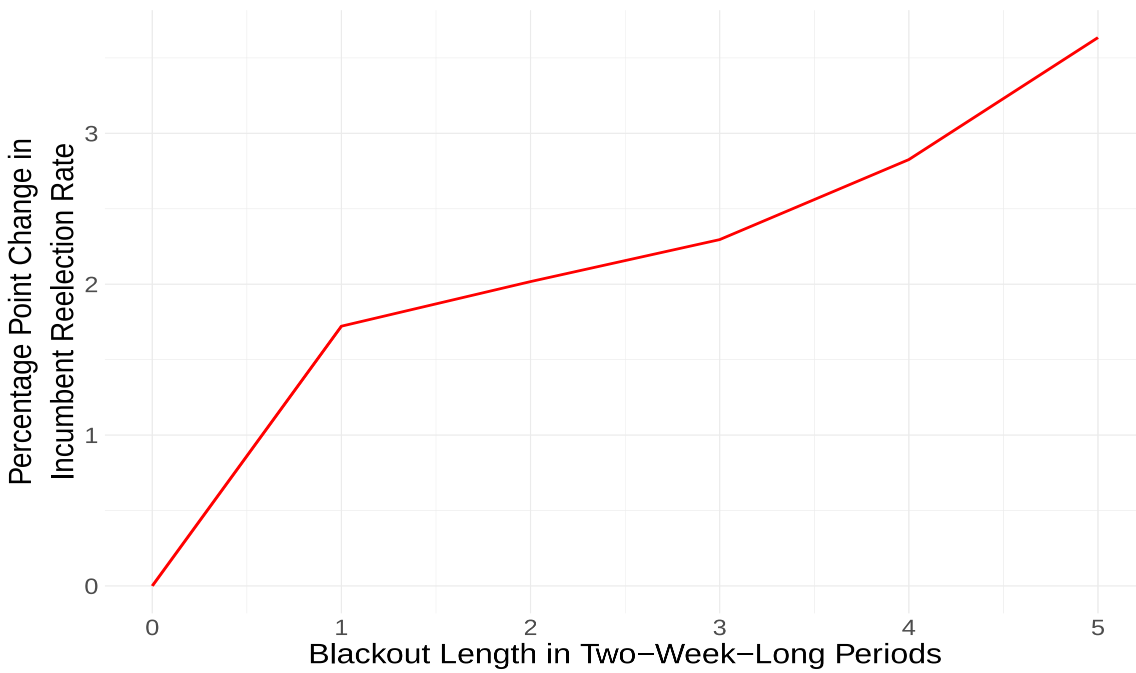
To gain further insights into the role negative advertising plays in Senate elections, I consider a hypothetical blackout period on negative advertising. Blackout periods are common restrictions in democracies worldwide that restrict when certain campaign activities, like advertising or the release of opinion polls, can take place.[19] Using the estimated model, I simulate 15,000 counterfactual elections with the same distribution of initial conditions as in the data where candidates can only air negative ads in the final weeks of the election. I repeat these simulations for blackouts that vary in duration from the final two weeks of the race to a complete ten week negativity ban.

Figures 4 and 5 show the effects of banning negativity at the end of the race on the incumbent reelection rate and the ideological extremeness of challengers who replace incumbents. Limiting negativity comes with a trade off between higher potential incumbent entrenchment and lower Congressional polarization. The incumbent reelection rate rises once challengers are unable to air their most

[19]Israel, for example, bans polling in the final five days of the election, and bans TV and radio ads outside of a roughly two-week block before election day. Advertising in Argentina can begin only sixty days before an election, while Australia and Brazil ban television and radio advertising in roughly the final three days of their elections.

effective ads. At the same time, the more ideologically extreme challengers who run less effective positive ads and rely most heavily on negative advertising are no longer elected.

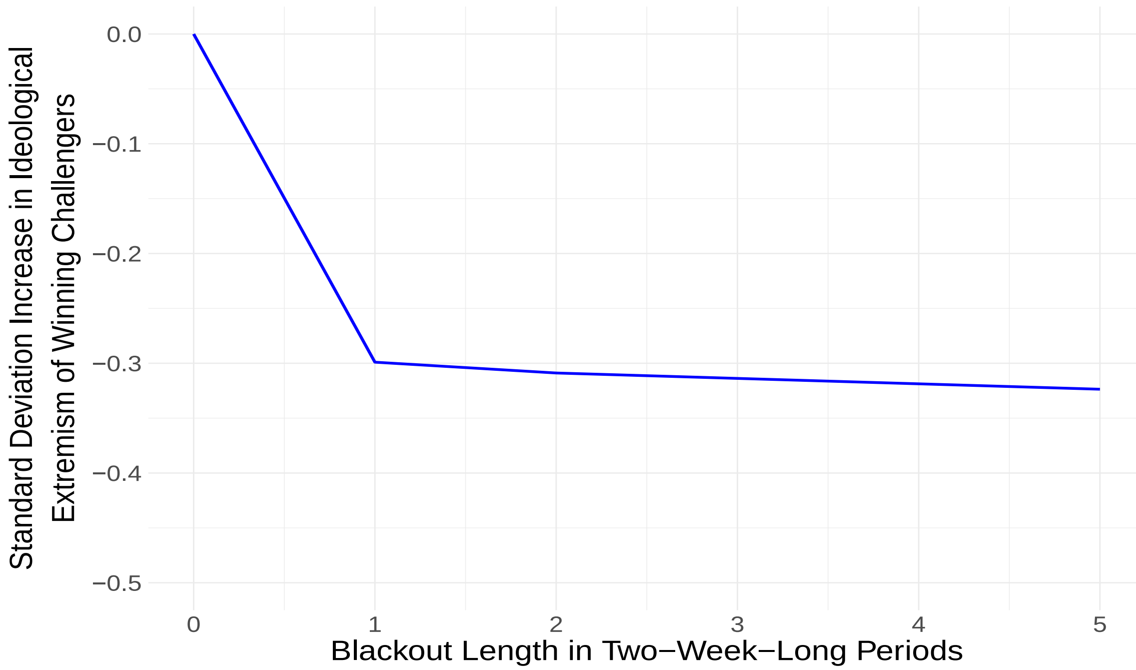
Figure 4: The Effect of Limiting Negativity on Incumbents' Reelection Rate



Note: Figure shows the relationship between blackout duration and the incumbent reelection rate. The incumbent reelection when there is no blackout is 73.8%. The X-axis records the number of consecutive two-week-long periods at the end of the race when negativity is banned.

Notably, banning negativity in the final two weeks of the election achieves virtually all of the gains in lowering the election rate of extremist challengers as a longer ban. Because the marginal returns of advertising are higher at the end of the race than at the beginning due to persistence in the polling transition equation, banning negativity in the final two weeks prevents challengers from running their most effective ads in the period when they are most effective. Combined with the fact that extreme challengers who unseat incumbents tend to do so by only narrow margins, this two-week-long ban on their most effective ads sinks their election prospects. The incumbent reelection rate, in contrast, continues to rise as the blackout duration increases, illustrating that a two-week-long blackout outperforms longer bans.

Figure 5: The Effect of Limiting Negativity on Winning Challengers' Ideology



Note: Figure shows the relationship between blackout duration and the ideological extremism of challengers who unseat incumbents. The X-axis records the number of consecutive two-week-long periods at the end of the race when negativity is banned.

That limiting negativity trades off higher incumbent entrenchment and less competitive elections with lower Congressional polarization has potentially important implications for public policy in light of the existing literature. On one hand, as discussed in the previous section, higher incumbent reelection rates may reflect worse representation for voters. Elections can only discipline representatives when they are competitive. In order for challengers to overcome incumbents' initial polling and fundraising advantages, they must be able to call out their opponent's shortcomings. Challengers are tasked with convincing voters that taking a chance on a electing a newcomer to the office is better than keeping their current representative, and it is hard to imagine how they can do so successfully without ever broadcasting the incumbents' flaws.

On the other hand, a significant rise in Congressional polarization over the last several decades has been documented in ideological measures based on roll call voting behavior (McCarty et al., 2006), congressional speech (Gentzkow et al., 2017), and campaign donations (Bonica, 2014). Policymakers and academics alike have voiced concern over the growing partisan divide among legislators, and polarization has been linked to negative policy outcomes, including legislative gridlock (Binder, 2003), income inequality (McCarty et al., 2016), and worse policy responses to financial crises (Mian et al., 2014). My results suggest that negative advertising was at least partly to blame for driving polarization in the U.S. Senate over the last two decades by facilitating ideologically extreme challengers in replacing more moderate incumbents.[20]

8 Conclusion

I have studied the impact of political advertising on the outcomes of competitive general elections to the U.S. Senate. Using uniquely detailed data on candidates' advertising and polling margins, I have documented evidence of candidates' strategic interactions and how a candidates' political experience, ideological platforms, and standings in the polls impact their advertising decisions. I then developed and estimated a dynamic game-theoretic model that is consistent with these facts.

Parameter estimates show that advertising *can* influence competitive election outcomes. However, I find that it is not just how much candidates spend that matters. The timing of when candidates advertise, how they divide their spending on positive and negative ads, and how these advertising decisions interact with both their own and their opponent's political backgrounds and ideological

[20]Related research by Moskowitz et al. (2020) has also linked the recent rise in polarization to the replacement of moderates by more extreme legislators, while Canen et al. (2020, 2021) show the importance of accounting for increased party discipline in decomposing the sources of polarization.

platforms all affect who wins the election.

Counterfactual policy experiments show that negative advertising plays an important role in keeping elections between incumbents and challengers competitive and helps challengers make up for their relative disadvantage on fundraising. However, negative advertising also facilitates the election of ideological extremists who rely most heavily on negative advertising to get elected. My results suggest more broadly that accounting for heterogeneity in ad content is key for understanding the role of advertising in competitive environments, and this finding may translate beyond political campaigns to commercial markets.

There are several limitations of my analysis that give ample scope for future research. First, counterfactual exercises are based on the distribution of initial conditions in the data that I take to be exogenous. In particular, I do not consider the possibility that counterfactual policy changes might impact the types of candidates who run for office. Future research might investigate how political competition impacts who runs for office.[21]

Second, my analysis also has nothing to say about how advertising influences voter turnout. Since the focus of this paper is on how advertising influences election outcomes, I only study how advertising shifts candidates' relative vote shares. Whether advertising influences vote shares by persuading swing voters, by mobilizing a candidate's base, or by demobilizing voters frustrated with excessive negativity is a separate question whose answer is still up for debate.[22]

Third, I only focus on two-candidate elections where relative vote shares determine the victor. In multi-candidate elections, such as U.S. primaries or in coun-

[21]While these issues have been modeled and examined theoretically (e.g., Osborne and Slivinski (1996); Besley and Coate (1997); Mattozzi and Merlo (2007)), much less on candidate selection has been documented empirically.

[22]For example, Ansolabehere et al. (1994) and Krupnikov (2011) find that negative advertising demobilizes voters, Goldstein and Freedman (2002) find that negative advertising mobilizes voters, and Finkel and Geer (1998), Ashworth and Clinton (2007), Krasno and Green (2008), and Spenkuch and Toniatti (2018) find no effect of advertising on turnout.

tries with many parties, negative advertising may create positive externalities for candidates who are not subject to the attack.[23] These externalities also extend to commercial markets, where firms care not only about their market share, but also the size of the market.[24] In principle, extending the model to accommodate more than two candidates is straightforward. In practice, it is well-known that the computational burden of solving the dynamic game grows rapidly with the number of players. Future research should investigate these issues more carefully.

Finally, because of data limitations, I am only able to model how candidates' advertising translates into voter support in a reduced form way. Existing work has examined various mechanisms through which ads influence voter behavior, including by persuading (e.g., Huber and Arceneaux, 2007; Lovett and Peress, 2015), by providing information (Kendall et al., 2015), and by indirectly driving viewers to engage with more political news (Canen and Martin, 2021). However, our understanding of how different types of ad content influence voter behavior remains incomplete. Advances in machine learning for audio, image, and video analysis make better understanding the mechanisms by which different types of ad content influence voter behavior an exciting area for future research.[25]

[23] Gandhi et al. (2016) and Galasso et al. (2020) study these externalities in U.S. Congressional primaries and Italian mayoral campaigns, respectively. Bernhardt and Ghosh (2020) study a related externality whereby excessive negative advertising in primaries can hurt the eventual primary winner in the general election.

[24] Anderson et al. (2015) study these externalities in pharmaceuticals markets.

[25] Constantinou (2022) and Le Pennec (2022), for example, use text data methods on ad transcripts to quantify additional dimensions of ad content, like whether the ad discusses policy positions or candidates' personal characteristics, and test predictions of novel theoretical models for when candidates should strategically air different types of ad content.

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A Additional Data Description

A.1 Coding Positive and Negative Ads

I follow Lovett and Shachar (2011) and categorize ads as positive or negative as follows. Each ad contains an indicator classifying it as positive, negative, or contrast. Contrast ads are then further classified as either primarily positive, primarily negative, or both positive and negative. Positive ads are defined as those ads classified as positive or primarily positive contrast, with the remaining ads defined as negative.

The reclassification of contrast ads as either positive or negative stems from Ken Goldstein, former director of the Wisconsin Advertising Project which preceded the Wesleyan Media Project. He makes the case that most positive information in contrast ads is overshadowed by larger quantities of potentially more memorable negative information. There is, moreover, a notion in the literature that any contrast ad is by definition a negative ad, since the intention of contrasting yourself with your opponent is to make yourself appear more favorable and your opponent appear less favorable. See Goldstein and Freedman (2002) for further discussion of the CMAG data and the coding of positive and negative ads.

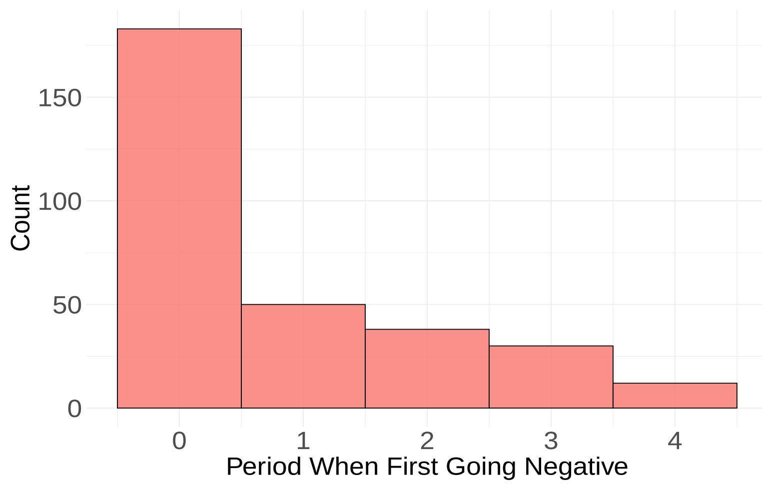
A.2 Converting Ad Spending into GRPs

The CMAG data does not contain GRPs, but GRP measures can be constructed based on the estimated ad expenditures contained in CMAG and prices per GRP, known as the cost per point (CPP), which come from SQAD. GRPs are computed by dividing ad expenditures by their corresponding CPPs.

A.3 Variation in Time to Going Negative

Figure 6 shows the variation in when candidates first air negative ads. Roughly half of the candidates go negative in the first period. 92 percent of the 340 candidates eventually go negative.

Figure 6: Variation in Time to Going Negative



Note: Histogram plots the frequency of how many candidates air negative ads for the first time in each period of the election. 313 of the total 340 candidates eventually air negative ads.

A.4 Trends in Ad Airings Instead of GRPs

Figure 7 shows the mean number of ads aired, rather than mean GRPs shown in 2, in each two-week-long period of the election. Comparing the two figures shows that the dynamic trends in negativity are robust to using either ad airings or GRPs as the measure of advertising intensity.

A.5 Prior Validation of DIME Scores

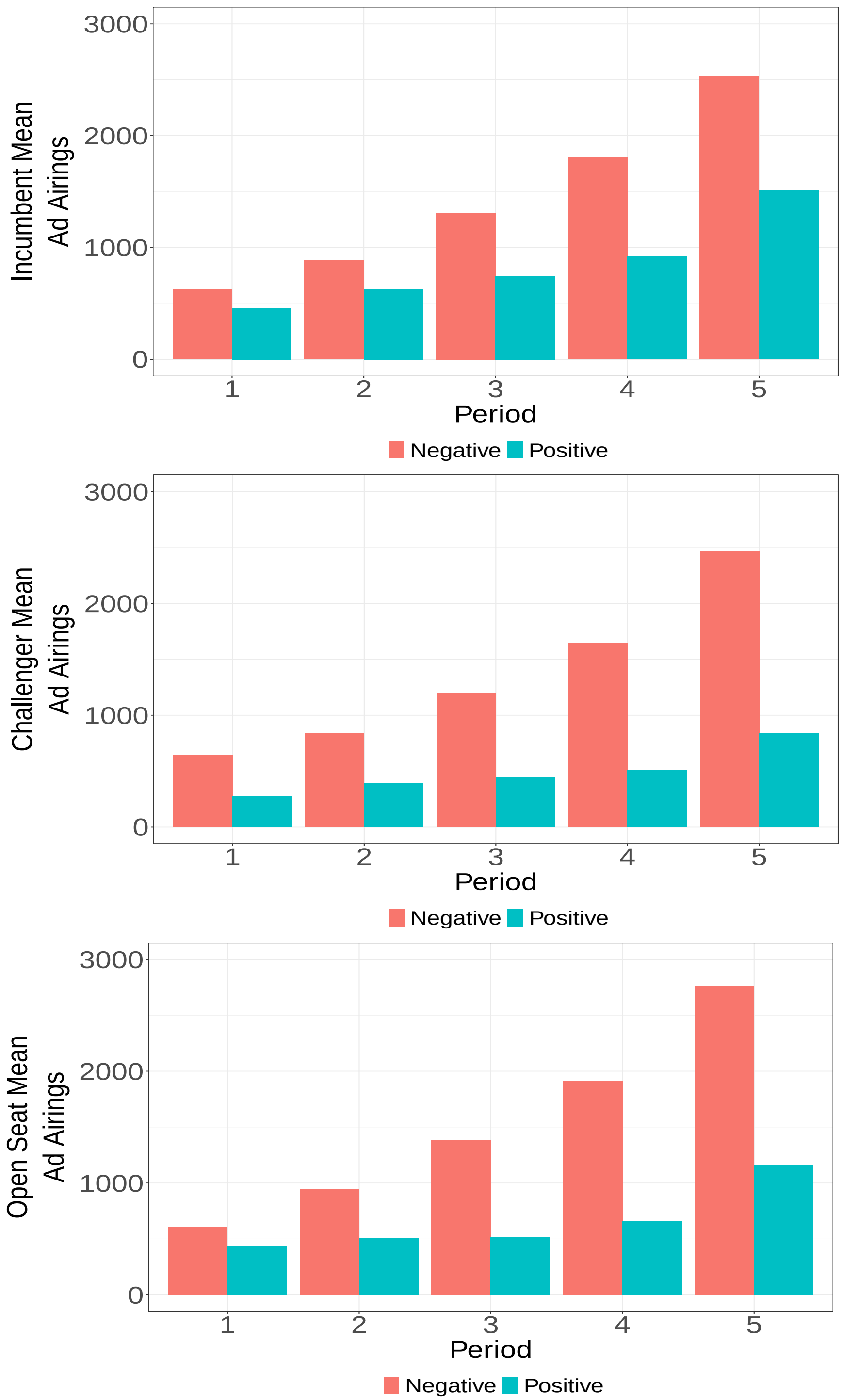
Campaign donation-based measures of ideology have been extensively validated across several studies spanning a variety of institutional settings and types of actors. For example, Bonica (2019) shows that it is possible to accurately forecast candidates' responses to policy items in the Congressional Campaign Election Study for a wide range of issues, both across and within party, using donation-based ideology measures. Bonica (2018) also shows that candidates' fundraising activity as non-incumbents is highly predictive of future roll-call voting behavior. Hall (2017) shows that previous ideology is an extremely strong predictor of current ideology, and Bonica (2014) demonstrated the ideological consistency in donors' contribution patterns and the robustness of DIME scores to strategic giving. A full compendium of validation results is available on Adam Bonica's website.

A.6 Sample of Elections

Because I only include races for which I can construct a time series of polling margins, and because highly non-competitive races were rarely or never polled during the election, my sample consists of more competitive races with higher spending and closer polling margins than the typical Senate race. In my sample, incumbents win reelection only 72 percent of the time, whereas the average Senate incumbent reelection rate between 2000 and 2018 was 86 percent according to data from OpenSecrets.org.[26] Moreover, around 15 percent of elections in my sample were decided within less than a 3 percentage point vote margin on election day.

[26] https://www.opensecrets.org/elections-overview/reelection-rates

Figure 7: Ad Airings Over the Election



Note: From top to bottom, the panels show the mean positive and negative number of ad airings by incumbents, challengers, and open seat candidates, respectively, in each two-week period of the election. Period one is the first period of the election. Period five ends on election day.

B Discretizing the State and Action Spaces

B.1 Applying Tauchen's (1986) Method

I discretize the the set of polling margins into 20 equispaced bins between -20 and 50 percentage points. I discretize the set of weekly ad expenditures into 20 equispaced bins between 0 and 30,000 GRPs. I discretize the set of negativity choices into 10 equispaced bins between 0 and 1.

After discretizing the set of polling margins, I must also discretize the transition equation. For a given vector of parameters θ governing the polls' law of motion, the Markov transition probability matrix induced by the choice of controls $a_t = (a_{Ct}, a_{It})$ can be computed following Tauchen's (1986) method for discretizing an AR(1) process:

1. Let $g_t \equiv g(y_t, a_t; \theta)$ denote the observable part of the law of motion that depends on the current state $x_t = (p_t, y_t)$ and controls so that the law of motion takes the form:
$$p_{t-1} = \rho p_t + g_t + \nu_t \tag{19}$$
where ν_t is an iid disturbance drawn from a mean zero normal distribution with variance σ_ν^2.

2. Let $\widetilde{p_t}$ denote the discrete-valued process that approximates the continuous-valued process, and let $\overline{p}_1 < \overline{p}_2 ... < \overline{p}_N$ denote the values $\widetilde{p_t}$ can take.

3. Let $w \equiv \overline{p}_j - \overline{p}_{j-1}$. Note that when the discrete values are equispaced, w is a constant independent of j and represents the width of the bin whose midpoint is one of the discrete values $\overline{p}_j$ that the polling spread can take. The transition probabilities from state i to state j are given below. For each

state i, if $1 < j < N$, set

$$\begin{aligned}
Pr(\widetilde{p}_{t+1} = \overline{p}^j | \widetilde{p}_t = \overline{p}^i) &= Pr\left(\overline{p}^j - \frac{w}{2} \leq \rho\overline{p}^i + g_t + \nu_t \leq \overline{p}^j + \frac{w}{2}\right) \\
&= \Phi\left(\frac{\overline{p}^j + \frac{w}{2} - \rho\overline{p}^i - g_t}{\sigma_\nu}\right) - \Phi\left(\frac{\overline{p}^j - \frac{w}{2} - \rho\overline{p}^i - g_t}{\sigma_\nu}\right)
\end{aligned}$$

otherwise,

$$Pr(\widetilde{p}_{t+1} = \overline{p}^1 | \widetilde{p}_t = \overline{p}^i) = \Phi\left(\frac{\overline{p}^1 + \frac{w}{2} - \rho\overline{p}^i - g_t}{\sigma_\nu}\right)$$

and

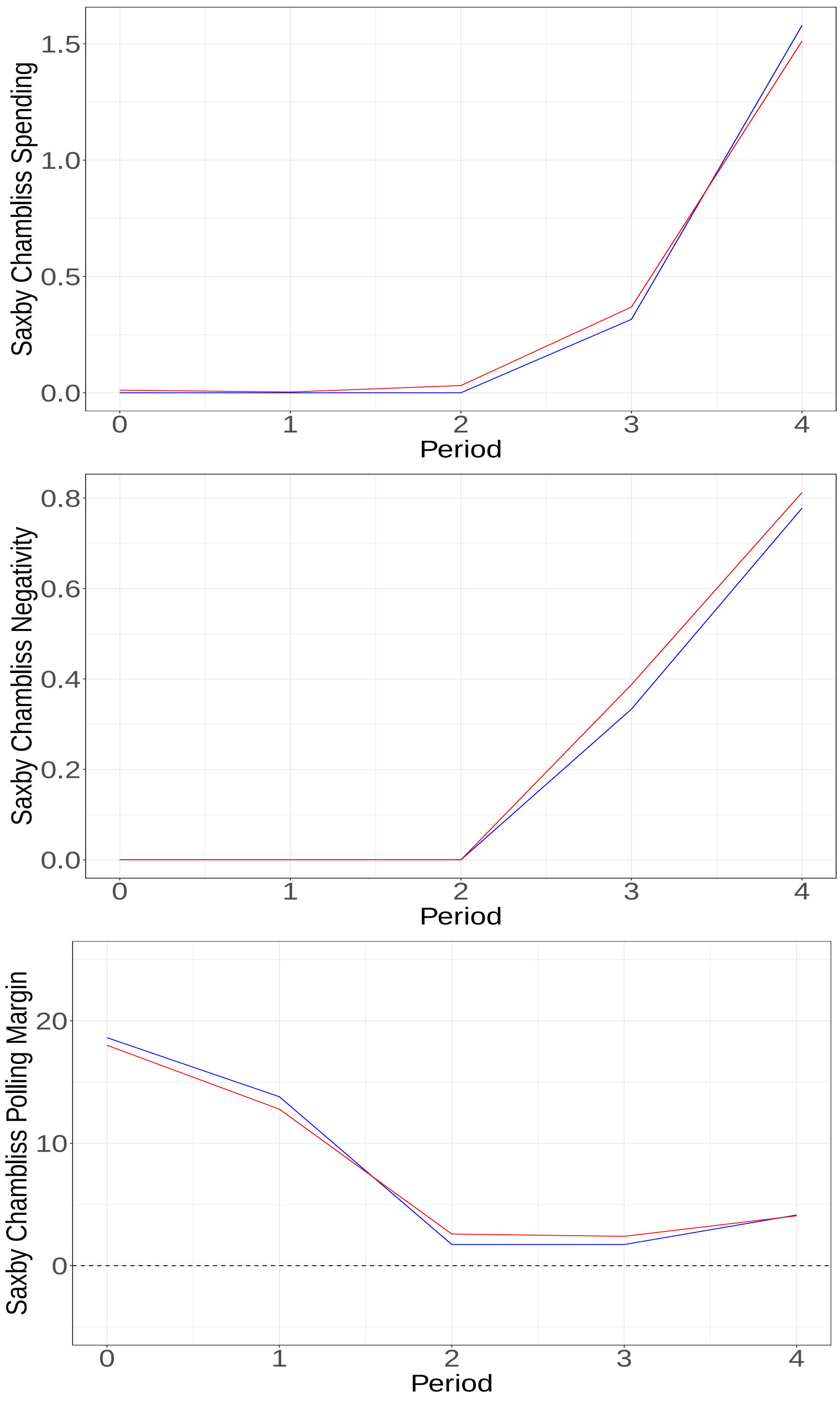
$$Pr(\widetilde{p}_{t+1} = \overline{p}^N | \widetilde{p}_t = \overline{p}^i) = \Phi\left(\frac{\overline{p}^N - \frac{w}{2} - \rho\overline{p}^i - g_t}{\sigma_\nu}\right)$$

where Φ is the standard normal CDF.

B.2 Discretization Fit

Figure 8 shows how the discretized spending, negativity, and polling margins compare with the real values in the data for an example candidate: Saxby Chambliss's 2008 Georgia Senate campaign. The discretization is sufficiently fine to capture the changes in Chambliss's advertising decisions and polling advantage throughout the election.

Figure 8: Discretization Fit



Note: From top to bottom, the panels show the discretized vs. real spending, negativity, and polling margins for Saxby Chambliss in the 2008 Georgia election. Discretized values are in blue. Real values are in red.

B.3 Robustness of Estimation to Discretization

Tables 6 and 7 compare the model estimates when increasing the number of bins for each discretized variable by 150 percent. The estimates are nearly identical when using a finer discretization.

Table 6: Preference Parameter Estimates and Asymptotic Standard Errors

Parameter	Description	Main Specification	Standard Error	Finer Discretization	Standard Error
κ_{C0}	Challenger Cost Intercept	0.60	0.11	0.61	0.12
κ_{C1}	Toss Up	-0.43	0.07	-0.42	0.08
κ_{C2}	Post-Citizens United	-0.33	0.08	-0.34	0.08
κ_{C3}	Open Race	-0.11	0.06	-0.12	0.07
κ_{I0}	Incumbent Cost Intercept	0.35	0.07	0.35	0.08
κ_{I1}	Toss Up	-0.27	0.05	-0.26	0.05
κ_{I2}	Post-Citizens United	-0.44	0.11	-0.45	0.11
κ_{I3}	Open Race	0.11	0.04	0.12	0.05
ω	Officeholder Benefits	3.10	0.81	3.21	0.85

Note: Table contains preference parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$).

Table 7: Transition Parameter Estimates and Asymptotic Standard Errors

Parameter	Description	Main Specification	Standard Error	Finer Discretization	Standard Error
β_{C10}	Negative Spending Intercept	1.13	0.09	1.15	0.11
β_{C11}	Incumbent Gone Negative × Incumbent Ideology	0.51	0.04	0.49	0.05
β_{C12}	Toss Up	0.22	0.03	0.24	0.05
β_{C13}	Open Seat	-0.09	0.05	-0.10	0.05
β_{I10}	Negative Spending Intercept	0.39	0.03	0.34	0.04
β_{I11}	Challenger Gone Negative × Challenger Ideology	0.47	0.07	0.44	0.07
β_{I12}	Toss Up	0.07	0.03	0.08	0.03
β_{I13}	Open Seat	0.11	0.04	0.12	0.05
β_{C20}	Positive Spending Intercept	0.63	0.08	0.61	0.09
β_{C21}	Open Seat	0.03	0.02	0.03	0.01
β_{C22}	Challenger Ideology	-0.43	0.09	-0.44	0.08
β_{I20}	Positive Spending Intercept	0.57	0.06	0.54	0.06
β_{I21}	Open Seat	-0.07	0.03	-0.06	0.04
β_{I22}	Incumbent Ideology	-0.14	0.15	-0.11	0.12
τ_{C1}	Challenger Power on Negative Spending	0.90	0.08	0.87	0.09
τ_{I1}	Incumbent Power on Negative Spending	0.67	0.07	0.65	0.06
ρ	Autoregressive Parameter	0.85	0.17	0.85	0.18
ϕ_0	Intercept	0.21	0.04	0.23	0.05
ϕ_1	Unemployment Rate	0.09	0.52	0.12	0.51
ϕ_2	Partisanship Index	0.48	0.21	0.46	0.20
σ_ν	Transition shock standard deviation	4.31	0.72	4.37	0.75

Note: Table contains transition parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$).

C Robustness to Timing Assumption

Tables 8 and 9 compare the model estimates when the incumbent is assumed to move first with the estimates when the challenger is assumed to move first. The estimates are quite similar regardless of which candidate is assumed to move first. This suggests that any artificial first-mover advantage resulting from assuming a deterministic ordering of moves is minimal in this context.

Table 8: Preference Parameter Estimates and Asymptotic Standard Errors

Parameter	Description	Incumbent Moves First Estimate	Standard Error	Challenger Moves First Estimate	Standard Error
κ_{C0}	Challenger Cost Intercept	0.60	0.11	0.55	0.12
κ_{C1}	Toss Up	-0.43	0.07	-0.40	0.09
κ_{C2}	Post-Citizens United	-0.33	0.08	-0.34	0.08
κ_{C3}	Open Race	-0.11	0.06	-0.12	0.07
κ_{I0}	Incumbent Cost Intercept	0.35	0.07	0.37	0.08
κ_{I1}	Toss Up	-0.27	0.05	-0.29	0.05
κ_{I2}	Post-Citizens United	-0.44	0.11	-0.46	0.12
κ_{I3}	Open Race	0.11	0.04	0.13	0.05
ω	Officeholder Benefits	3.10	0.81	3.41	0.92

Note: Table contains preference parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$).

Table 9: Transition Parameter Estimates and Asymptotic Standard Errors

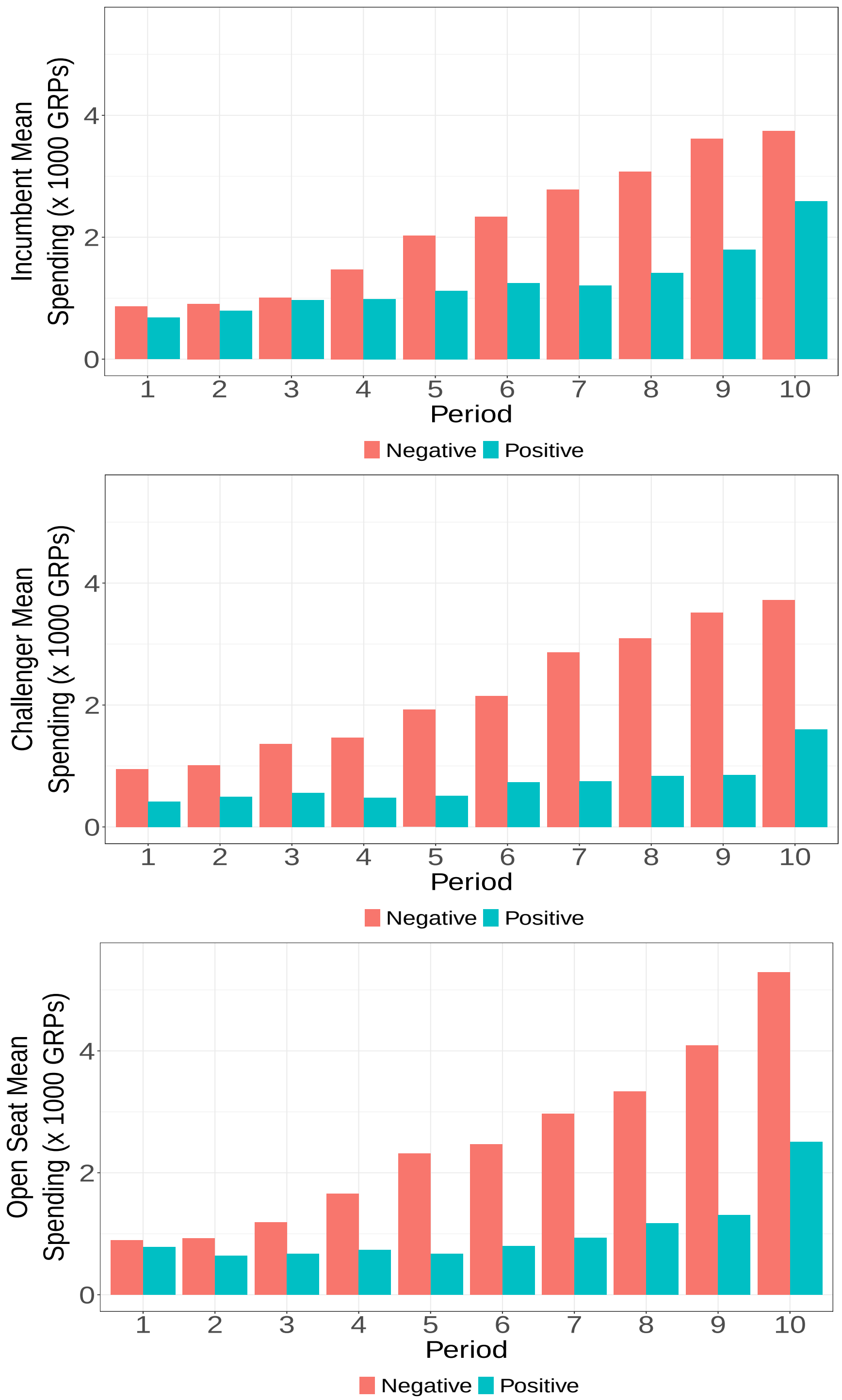
Parameter	Description	Incumbent Moves First Estimate	Standard Error	Challenger Moves First Estimate	Standard Error
β_{C10}	Negative Spending Intercept	1.13	0.09	1.19	0.11
β_{C11}	Incumbent Gone Negative $\times$ Incumbent Ideology	0.51	0.04	0.54	0.05
β_{C12}	Toss Up	0.22	0.03	0.20	0.05
β_{C13}	Open Seat	-0.09	0.05	-0.11	0.05
β_{I10}	Negative Spending Intercept	0.39	0.03	0.31	0.04
β_{I11}	Challenger Gone Negative $\times$ Challenger Ideology	0.47	0.07	0.38	0.06
β_{I12}	Toss Up	0.07	0.03	0.08	0.03
β_{I13}	Open Seat	0.11	0.04	0.13	0.05
β_{C20}	Positive Spending Intercept	0.63	0.08	0.59	0.09
β_{C21}	Open Seat	0.03	0.02	0.02	0.01
β_{C22}	Challenger Ideology	-0.43	0.09	-0.41	0.07
β_{I20}	Positive Spending Intercept	0.57	0.06	0.54	0.05
β_{I21}	Open Seat	-0.07	0.03	-0.05	0.04
β_{I22}	Incumbent Ideology	-0.14	0.15	0.04	0.11
τ_{C1}	Challenger Power on Negative Spending	0.90	0.08	0.84	0.08
τ_{I1}	Incumbent Power on Negative Spending	0.67	0.07	0.65	0.06
ρ	Autoregressive Parameter	0.85	0.17	0.82	0.19
ϕ_0	Intercept	0.21	0.04	0.19	0.04
ϕ_1	Unemployment Rate	0.09	0.52	0.16	0.49
ϕ_2	Partisanship Index	0.48	0.21	0.39	0.20
σ_ν	Transition shock standard deviation	4.31	0.72	4.88	0.69

Note: Table contains transition parameter estimates and their asymptotic standard errors. Ad spending is in GRPs ($\times 10,000$).

D Weekly Spending and Negativity Dynamics

Figure 9 shows that the weekly dynamics of ad spending and negativity are the same as for the bi-weekly dynamics.

Figure 9: Weekly Ad Spending Over the Election



Note: From top to bottom, the panels show the mean spending on positive and negative ads by incumbents, challengers, and open seat candidates, respectively, in each week of the election. Period one is the first period of the election. Period ten ends on election day.

E Within-Sample Model Fit

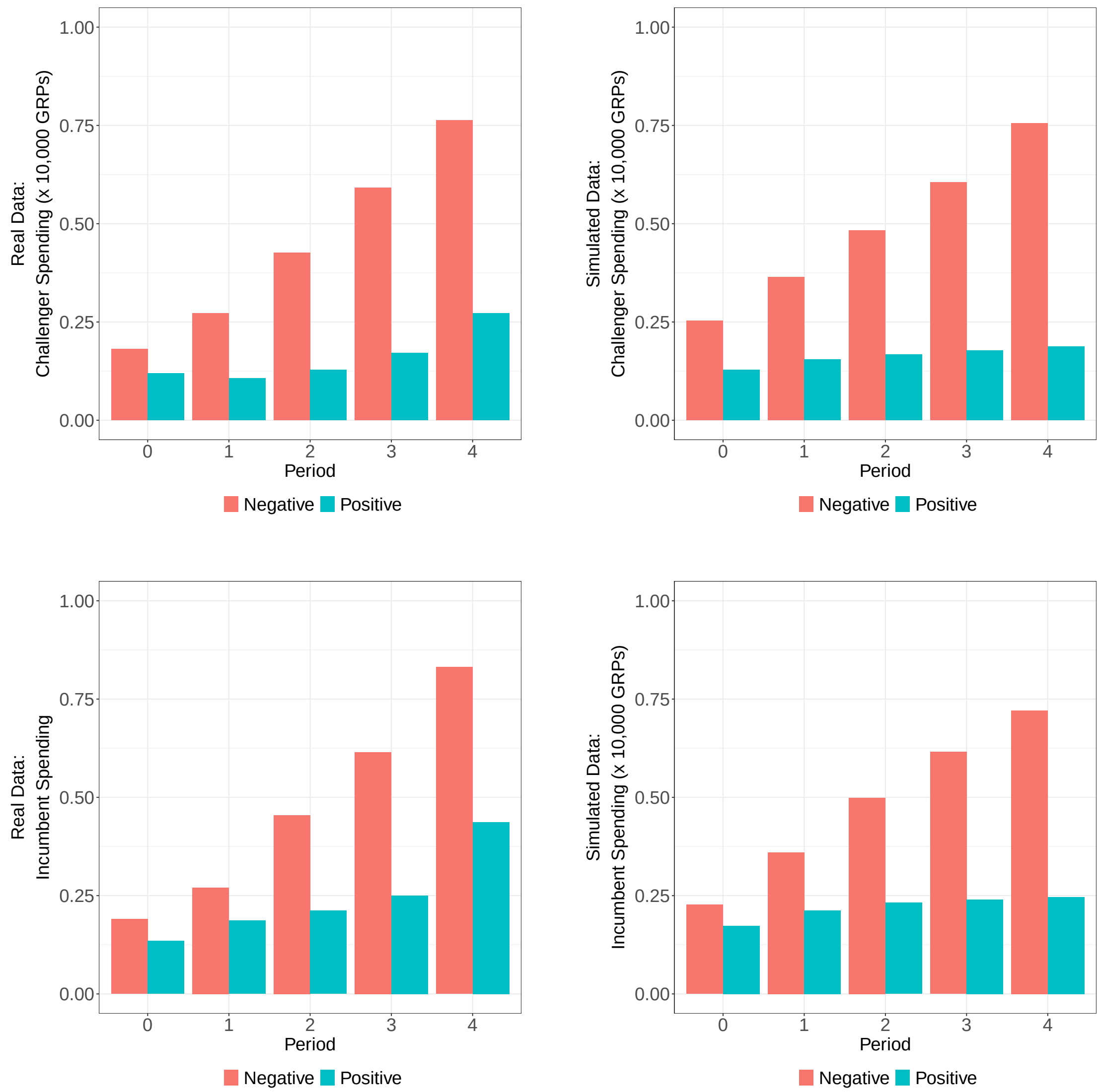
To evaluate how the model fits the data, I compare different moments related to candidates' advertising decisions and election outcomes in the real data with data simulated from the estimated model. First, the incumbent reelection rate in the simulated data is 73.8 percent, compared with 72.2 percent in the real data. Second, Figure 10 compares the average spending on positive and negative ads in the simulated data to the real data. Finally, Table 10 compares the fraction of candidates who air their first negative ads during each period of the election in the simulated data to the real data.

Table 10: Comparing Real and Simulated Going Negative Timing Decisions

	Percent of Candidates who Go Negative in Period t	
	Real Data	Simulated Data
Period 0	53.82	57.11
Period 1	14.70	16.49
Period 2	11.17	10.65
Period 3	8.82	7.21
Period 4	3.52	3.95

Note: Table compares the percent of candidates who first air negative ads in each two-week period of the election in the real data with data simulated from the estimated model.

Figure 10: Comparing Real and Simulated Spending and Negativity Dynamics



Note: Figure compares the average spending on positive and negative ads in the real data with data simulated from the estimated model. As defined in the model and estimation, incumbents are the candidates whose party most recently held the seat.